

“We are all your students, Mr. Cartan”

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At the end of June, 1965, at the (then) new mathematics library of the Ecole normale supérieure in Paris (ENS), a party was held in honor of Henri Cartan. He was leaving his position of “directeur des études mathématiques” at ENS, after 25 years of service. Many mathematicians gathered on the occasion of this farewell. Cartan, in one of his customary understatements, commented: “I asked to extend the invitation to my former pupils, and I see so many people. . .”. To which, Vladimir Arnold, visiting France for the first time, and in his standard wit, answered: “But . . . Mr Cartan . . . in Moscow as well as in Paris, we are all your students”. Cartan was extremely pleased.

Of the two great creations of the revolutionary Convention (1793), if Ecole Polytechnique is, in the words of Arthur Wightman, a “peculiar combination of West Point and M.I.T.”, Ecole normale is properly the highest in a network of teacher’s colleges.

When I was admitted at ENS in 1950, the mathematics department consisted of a rich and spacious library, with two offices at the entrance gate, shared by the professor, his teaching assistant (a “caïman”¹ in our student’s slang), and the librarian. The professor was Henri Cartan, his caïman was Jean Frenkel, a veteran from WWII², the librarian was the wife of the head librarian of the school, named Madame Martin. There were two more classrooms, called E and F, with the standard joke that every event was announced to take place in room E *or* F. Everything took place in these two small rooms: the Bourbaki Seminar, then in its ascending phase, the standard lectures for the students, the entrance examination.

Mathematics students were allowed only three years for the standard curriculum, while our friends in the other departments were given four years.

¹= alligator

²and a Jew who left France in 1942 to join de Gaulle’s Free French Forces.

Obtaining a fellowship for a supplementary year (to begin a doctoral thesis) was quite *hasardous*.

Cartan was everywhere. He himself taught the three years. In the first year, he supplemented – and modernized – the Sorbonne course on Calculus. In this first year, we were supposed to follow at the Sorbonne the main courses: Calcul différentiel et intégral (Calculus), General physics and Classical mechanics. These courses were quite old-fashioned.

In the second year, in place of a master thesis, the mathematics students were required to follow an advanced course: Algebra (or Analysis, or Geometry) “supérieure”. But the main feat was for us Cartan’s course called “cours aux carrés³”. Having married during the summer 1951, to the great dismay of Cartan, rather conservative in social habits, I was less faithful a student, but I remember memorizing hastily Erdős-Selberg’s proof of the prime number theorem⁴ as part of “algèbre supérieure”. Cartan impressed me very much with a course on potential theory, using the new tool of distributions and a simplified version of Sobolev spaces known as Dirichlet space.

For many years, the main academic obligation for the students of ENS was to submit to a national competition called “Agrégation”. Officially, this is a qualifying examination for the profession of high school teacher, in an extended sense, from junior high school to the elite undergraduate program known as “classes préparatoires”.

In our third year, still under the guidance of Cartan, we prepared for the “agrégation”. He read very carefully our memoirs, twice a month, and trained us for the oral examination. I hated the completely old-fashioned curriculum, and was impatient to return to more serious – and modern – matters. But, preparing for the oral examination – a course in which one *teaches how to teach* – is a fruitful exercise, and I was later happy to teach such a course.

I remember a somewhat embarrassing episode. I was supposed to explain the standard result that a function, whose derivative exists everywhere and is identically zero, is a constant. I prepared carefully during the Christmas vacation, using a proof in Bourbaki allowing some exceptional points. When it was my turn, I explained to the class that I intended to “improve” Bourbaki’s proof by allowing an exceptional set of measure 0. I remember Cartan

³Second Year students

⁴A so-called “elementary proof”, a tricky and uninformative extension of Čebičev reasonings!

commenting: “Now, listen carefully, Cartier is going to prove his first theorem!” I understood, by his tone, that something was wrong. I often taught Lebesgue’s integral, and never slipped again!

But the real event was Cartan’s seminar. I vividly remember my first class at ENS, the second Monday of November 1950. According to what became the weekly routine, I attended in the morning Cartan’s class for the “conscrits” (freshmen), then at 2:30 p.m, his seminar, and in the late afternoon, we were supposed to learn from the craftsmen the art of cutting and assembling metal, wood, glass, for a possible carrier in experimental physics. I was not very good at that, and learned just enough to help my friends preparing fantastic illuminations for the school’s night in the next spring.

Cartan had an indomitable curiosity and openness. He was also very tenacious, and his friends often called him “mosquito” for his insistence to bite. He was a quite rigorous huguenot, and I soon learned that he was during the summer the organ player for a small protestant community in Die, where the mourning took place after his death. He could be quite formal, but in a very british way, compatible with wit. I remember my first visit to his just reunited family (in their summer home). His daughter was wearing trousers for the first time, and in a very formal tone he asked his wife: “My dear friend, have you noticed the dress of your daughter?” Despite this formality, he had an open mind: always well-dressed, he never blamed me for my sloppiness, intended to provoke my elders!

A glance at the table of contents of his seminar shows an unusual diversity of topics: algebraic topology, sheaf theory, several complex variables, automorphic functions, algebraic geometry, index theorem, . . . He gave himself many of the lectures, the other speakers spent long hours with him for preparation, and he wrote (and typed) himself a large part of the proceedings.

After a few weeks, during which I understood nothing, I had acquired an elder brother – Jean-Pierre Serre – and a benevolent uncle – Samuel Eilenberg –, a protestant and a Polish Jew, well in line with my roots in Mittel – Europa. I made an acquaintance with all – or almost all – French mathematicians. At that time, everything in mathematics was in Paris, around Cartan, with two extensions in Nancy⁵ – where Dieudonné, Delsarte, Schwartz maintained Bourbaki’s spirit – and in “der Reichsuniversität Straßburg” with Ehresmann

⁵The other arm of Bourbaki was in Chicago, hence the famous series of Nancago mathematics books.

(for a while a Bourbaki accomplice). The glorious generation – Hadamard, Borel, Fréchet, Paul Lévy, Elie Cartan⁶ – was very old or already gone, and the ambitious youth gathered around Cartan. By his central position at the ENS, Cartan was more or less the thesis advisor of everyone (at least formally).

From Eilenberg, in the academic year 1950-51, we heard a long series of lectures in which he developed an axiomatic theory of group homology, as a model for his well-known axiomatisation of algebraic topology, in his book with Steenrod. This series was to be followed by a similar attempt towards the cohomology of sheaves – a much more difficult subject, whose completion was one of the first major achievements by Grothendieck.

Three times a year, the same group of people attended the Bourbaki Seminar in its ascending phase⁷. There we learned about Weil’s proof of the Riemann hypothesis in the case of function fields, Zariski’s work (as reported by P. Samuel), Koszul’s thesis about the homology of Lie algebras, the work of Petrowsky in partial differential equations and of Gelfand in the theory of group representations, . . . all the hot subjects in mathematics. Moreover, the supposedly secret drafts of forthcoming Bourbaki volumes were freely circulated by Serre, Cartan, . . . Among the closest friends of Cartan were André Weil, who visited France every summer and gave a series of lectures at the ENS in the winter 1951 (about adeles and ideles), Armand Borel, who introduced Cartan to Leray’s work on sheaves, Claude Chevalley, who became a professor at La Sorbonne after 1954 (and one year in Japan).

French mathematics were at a turning point. The undergraduate curriculum (even in its enhanced form for the “classes préparatoires”) was a mix of coordinate geometry, synthetic geometry (based on the “theorem” that every one-to-one correspondance between the points of a projective line is given by a Möbius transformation⁸), differential calculus with applications to geometry and kinematics. The foundations were sometimes shaky and there was hardly any hint of groups of transformations (in geometry) and

⁶Father of Henri Cartan!

⁷To reach its peak of fame and attendance from 1970 to 1990, followed by a slow decline.

⁸Laurent Schwartz embarrassed his teacher with the counter-example of going to the complex conjugate!

the use of matrices was ignored or not advised⁹. The good teachers dared to give the foundations of the real number system, but in the absence of set-theoretical terminology, the exposition was quite obscure. In the land of Lebesgue, hardly any mention was made of Lebesgue integral, and we had to learn Lie groups from the thesis of Elie Cartan or in Pontrjagin!

By a sequence of well planned steps, Cartan made General Bourbaki win! He managed to hire the ambitious youth at La Sorbonne: Schwartz, Choquet, Dixmier, Godement, Chevalley and in 1957 the takeover was complete (except for André Weil, who was never forgiven for his refusal to be drafted in 1939, at the beginning of WWII!). The curriculum was deeply renovated, and the textbook of textbooks became Bourbaki (whose golden age extends from 1950 to 1975!). The forceful gesticulations of Dieudonné, as well as the power of persuasion of Choquet and Lichnerowicz (both not members of Bourbaki) convinced everyone to worship General Topology, Linear Algebra, Functional Analysis and Group Theory. Henri Cartan was always a moderate in this debate, and never threw the baby with the bath tub. He made one major mistake, to discard classical mechanics, with this comment: “The teaching of classical mechanics (in France) is very poor – which was true! –, and for the physicist, only Quantum Mechanics matters”. We know better, as no one can understand Quantum Mechanics without a thorough acquaintance with its newtonian (or hamiltonian) version. One of the reasons for this mistake was the almost complete ignorance of the challenges of mathematical physics by the members of the Bourbaki *coterie*. It is also to be said that the teaching of physics in France was even more backward than in mathematics. One example: the first serious course on Quantum Mechanics was given in 1964, not in Paris, but in the National Center for Nuclear Physics (at Saclay).

So around 1960, Bourbaki had won, in main part due to the efforts of Cartan, helped by the political know-how of Schwartz. There followed the dubious episode of the so-called “modern math”, or the failed attempt to use Bourbaki as a textbook for kindergarten! The fiercest proponents were ultra-zealous disciples of Bourbaki, not of the same mathematical caliber, helped by an attraction towards abstractness, an integral part of the then prevalent fashions (in art and elsewhere).

For about 15 years, French mathematics was ruled by two enlightened

⁹Ostensibly, because there was supposedly no agreement about the product rule: lines by columns or columns by lines!

despots: Cartan and Schwartz, controlling between them most of the academic world. Nominally, every doctoral student in mathematics had one of them as thesis advisor, and they suggested research topics. Cartan's way was more open and deep, but everyone benefitted from their valued advice. Committee work was reduced to a couple of days off for our masters, where everything was settled, and in most cases, in an optimal way.

Under this reign, what was the fate of the young French mathematician? Let me take myself as an example. At the end of my first year at ENS, I was at pain choosing for the future; I was attracted by philosophy and (experimental) physics as well as by mathematics. The advice of Althusser, the professor of philosophy, was a warm support, with a fatherly caution about the degrees to be earned. Yves Rocard, head of the physics department, whose son Michel was later a prime minister of France, wanted to recruit me to build with him the French atom bomb, to be exploded ten years later. Cartan, following the advice of Eilenberg, invited me to one of those secret meetings of Bourbaki! My fate was decided.

A few years later, Cartan turned me down for the Princeton fellowship (awarded later on to Douady) on the excuse that this didn't fit a married man¹⁰ (he was not especially happy about my early marriage!). Then, he turned me down for the position of "caïman" at the ENS, explaining to me later that I would have sacrificed my research work because of my involvement in teaching (he was perfectly right!).

A deeper interference came later in 1961. After completing my thesis, and two years of postdoc at the Institute for Advanced Study in Princeton, and almost three years of draft at the time of the war of independence of Algeria, it was time for me to apply for a professorship (I was almost 30!). For each position I applied to, I got the same answer: "Why should you come here, since Cartan says that you have been appointed at Strasbourg (where I didn't apply!)" There was no way out, I tried to rebel and told Cartan I would stay only two years in Strasbourg – no more. He laughed and I spent willingly ten years there, in the opinion of my wife, our best years! He later gave me an explanation. He was enormously interested in bringing back together French and German mathematicians, as witnessed by his long-lasting friendship with Behnke and Hirzebruch. In Cartan's opinion, I was the best fit, in the young generation, to work in this direction. He was

¹⁰But not much later, I was invited for two years at the Institute for Advanced Study, where my wife was welcomed!

right and I immensely enjoyed working for the *Versöhnung*¹¹ with the help of my German friends Dold and Puppe. I was from that time on as convinced as he was of the necessity of a Federal Europe (still in the making), well in line with the thoughts of my mother, who explained to me, when I was five, that I should see the day of the United States of Europe.

Some years later in 1974, I didn't yield to his pressure to join Orsay University, but it was already another time, in the aftermath of the 1968 student revolution ... Grothendieck was gone already ...

Thank you, Henri Cartan, for your fatherly influence on me! It made me grow up!

¹¹Mutual forgiveness, in German!