Spatial variation of fundamental couplings and Lunar Laser Ranging

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Décembre 2011

IHES/P/11/30

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(Dated: 15 April 2011)

If the fundamental constants of nature have a cosmic spatial variation, there will in general be extra forces with a preferred direction in space which violate the equivalence principle. We show that the millimeter-precision Apache Point Observatory Lunar Laser-ranging Operation provides a very sensitive probe of such variation that has the capability of detecting a cosmic gradient of the ratio between the quark masses and the strong interaction scale at the level $\nabla \ln(m_{\rm quark}/\Lambda_{QCD}) \sim 2.6 \times 10^{-6}$ Glyr^{-1}, which is comparable to the cosmic gradients suggested by the recently reported measurements of Webb et al. We also point out the capability of presently planned improved equivalence principle tests, at the $\Delta g/g \lesssim 10^{-17}$ level, to probe similar cosmic gradients.

arXiv:1104.4872v1 [gr-qc] 26 Apr 2011

1. INTRODUCTION

Within many extensions of the Standard Model, the parameters of our fundamental theory need not be universally constant but may vary in space and time. The search for such variations provides important constraints on such theories. Recently, Webb et al. [1] have reported evidence for a non-zero spatial variation of the fine structure constant α. Parameterizing the variation of α by a dipole gradient

$$\frac{\alpha(\mathbf{x})}{\bar{\alpha}} = 1 + B_\alpha \,\hat{\mathbf{z}}_\alpha \cdot \mathbf{x} \tag{1}$$

they find evidence, at the $4.2\,\sigma$ level, for a slope parameter

$$B_\alpha = (1.10 \pm 0.25) \times 10^{-6} \text{ Glyr}^{-1} \tag{2}$$

relative to the unit direction $\hat{\mathbf{z}}_\alpha$ of right ascension $\alpha = 17.4 \pm 0.6$ hours and declination $\delta = -58 \pm 6$ degrees. In addition, Berengut et al. [2] found weak indications for the existence of a gradient of the electron to proton mass ratio $\mu \equiv m_e/m_p$ in the same direction $\hat{\mathbf{z}}_\mu = \hat{\mathbf{z}}_\alpha$, with slope

$$B_\mu = (2.6 \pm 1.3) \times 10^{-6} \text{ Glyr}^{-1}\,. \tag{3}$$

Other spatial gradients are much more weakly tested. For example, Donoghue and Donoghue [3] have used the spatial constancy of the first acoustic peak in the Cosmic Microwave Background to bound a possible variation in the cosmological constant (or generalized dark energy) at the level of an analogous slope parameter

$$B_\Lambda < 0.91 \times 10^{-2} \text{ Glyr}^{-1} \tag{4}$$

at the 95% confidence level.

Because the masses of all the elements depend on the parameters of the Standard Model, a gradient in one of these parameters will lead to a force (as noted long ago by Dicke [4]). Using the fine structure constant as an example, the dependence on α of the total mass-energy of system A,

$$E_A(\alpha) = c^2 M_A(\alpha) \ \ , \tag{5}$$

implies that a spatial gradient $\nabla\alpha$ of α , will lead to a force

$$\mathbf{F} = -\nabla E_A(\alpha) = -c^2 \frac{\partial M_A}{\partial \alpha} \nabla\alpha \ \ , \tag{6}$$

If we introduce the following dimensionless effective "charge" associated to the α dependence,

$$Q_\alpha(A) = \frac{\alpha}{M_A} \frac{\partial M_A}{\partial \alpha} \tag{7}$$

and parameterize the gradient of α by a slope and a unit direction as in Eq. (1), $\nabla\alpha/\alpha = B_\alpha \,\hat{\mathbf{z}}_\alpha$, the above force reads

$$\mathbf{F}_A = -Q_\alpha(A) \ M_A \ B_\alpha c^2 \ \hat{\mathbf{z}}_\alpha\,. \tag{8}$$

If we now consider the dependence of the total mass-energy $M_A c^2$ (in units of the Planck mass) of system A on the various dimensionless ratios (or coupling constants) $r_i = \alpha, \mu, m_{\rm quark}/m_p, \ldots$ entering physics at energy scales $\lesssim m_p c^2$, and if we assume the existence of (fractional) spatial gradients $\nabla \ln r_i = B_{r_i} \,\hat{\mathbf{z}}_{r_i}$ of the various dimensionless ratios, we see that body A will be submitted to an external *acceleration*, $\mathbf{g}_A$, of the form

$$\mathbf{g}_A = \frac{\mathbf{F}_A}{M_A} = -\sum_i Q_{r_i}(A) \ B_{r_i} c^2 \ \hat{\mathbf{z}}_{r_i} \tag{9}$$

where $Q_{r_i} = Q_\alpha, Q_\mu, \ldots$ are the various dimensionless effective "charges" associated to the dependence of the mass on the various ratios (or coupling constants), namely

$$Q_{r_i}(A) \equiv \frac{r_i}{M_A} \frac{\partial M_A}{\partial r_i} = \frac{\partial \ln(M_A/M_P)}{\partial \ln r_i} \tag{10}$$

In the second form of the definition of Q_{r_i} we have recalled that M_A is to be expressed in units of the Planck mass M_P. [This corresponds to working in the "Einstein conformal frame", where Newton's constant is held fixed.]

If the various effective charges $Q_{r_i}(A)$ were independent of the considered body A, the result would be an unobservable (gravity-like) uniform free fall with a universal acceleration $\mathbf{g}_0 = \mathbf{g}_A = \mathbf{g}_B$ in a direction given by an average of the various gradients $\nabla \ln r_i = B_{r_i}\,\hat{\mathbf{z}}_{r_i}$. However, composition dependence of (at least one of) the various charges, e.g. $Q_\alpha(A) - Q_\alpha(B) \neq 0$, or $Q_\mu(A) - Q_\mu(B) \neq 0$, will lead to differential accelerations $\mathbf{g}_A - \mathbf{g}_B \neq 0$ and locally observable effects. Recently, we have studied [5, 6] the composition dependence of the effective charges $Q_{r_i}(A)$ corresponding to a complete set of dimensionless ratios entering low-energy physics, namely $r_0 = \Lambda_{QCD}/M_P$, and

$$r_i = \alpha, m_u/\Lambda_{QCD}, m_d/\Lambda_{QCD}, m_e/\Lambda_{QCD}\,, \qquad (11)$$

where m_u, m_d are the masses of the light quarks.[1] Here, we separated the ratio $r_0 = \Lambda_{QCD}/M_P$, the dependence on which leads to composition-independent effects. We shall recall below our explicit results for the various charges $Q_{r_i}(A)$ $(i \neq 0)$.

Each slope parameter B_{r_i} defines a corresponding acceleration $B_{r_i}c^2$, which enters the total acceleration (9), multiplied by the corresponding dimensionless effective charge $Q_{r_i}(A)$. For instance, the α gradient (2) reported by Webb et al. [1] corresponds (using $c/1\text{yr}= 950\,\text{cm/s}^2$) to the acceleration level

$$B_\alpha c^2 = (1.05 \pm 0.24) \times 10^{-12}\,\text{cm/s}^2 \qquad (12)$$

while the acceleration level corresponding to the μ gradient suggested by Berengut et al. [2] is

$$B_\mu c^2 = (2.5 \pm 1.2) \times 10^{-12}\,\text{cm/s}^2 \qquad (13)$$

The aim of this paper is to point out that the EP-violating effects of spatial gradients of α and of the mean quark-mass ratio[2] $r_{\hat{m}} = \hat{m}/\Lambda_{QCD}$ (with $\hat{m} \equiv (m_u{+}m_d)/2$) at the levels Eqs. (12), (13) generate signals in the ranging to the Moon, which have a specific time structure, and an amplitude which seems large enough to be detectable by the recently started millimeter-precision Apache Point Observatory Lunar Laser-ranging Operation (APOLLO) [7, 8]. [See [9] for the results obtained from the pre-APOLLO Lunar Laser Ranging (LLR) experiments.] We also describe the weaker bounds on spatial gradients obtained by present laboratory-based experiments [10], and indicate that planned EP experiments at the $\Delta g/g \lesssim 10^{-17}$ level will probe cosmic gradients at the levels of Eqs. (12), (13).

Let us here emphasize the difference in outlook between our previous work, and the present study. In Refs. [5, 6], we were considering the case where the spatial or temporal variation of a dimensionless parameter indicates the existence of a field, say φ, which carries the spacetime dependence, and we were considering the violations of the "weak version" of the Equivalence Principle (EP), i.e. composition-dependent accelerations of body A, mediated by the coupling of φ to *local* matter distributions. As a consequence, the locally observable EP-violating effects depended on the product of it two φ coupling strengths, say $\alpha_A\,\alpha_E$, where

$$\alpha_A = \partial \ln(M_A(\varphi)/M_P)/\partial \ln \varphi$$

measures the coupling of φ to body A, and $\alpha_E = \partial \ln(M_E(\varphi)/M_P)/\partial \ln \varphi$ its coupling to an external "source" body E (which could be the Earth, the Sun, or some laboratory source). However, we had normalized the definition of the fundamental couplings d_{r_i} of the "dilaton" field φ, as they enter the low-energy Lagrangian, so that we could write each α_A in the specific form

$$\alpha_A = d_{r_0} + \sum_{i\neq 0} d_{r_i} Q_{r_i}(A) \qquad (14)$$

exhibiting a simple factorization between the fundamental dilaton couplings d_{r_0}, d_{r_i}, and the phenomenological effective charges defined in Eq. (10) above. Note that there is no composition-dependent charge associated to the coupling to $r_0 = \Lambda_{QCD}/M_P$, or, said differently, the charge $Q_{r_0}(A)$ associated to $r_0 = \Lambda_{QCD}/M_P$ is simply $Q_{r_0}(A) \equiv 1$ because, as the mass M_A can be written as the product of the hadronic mass scale Λ_{QCD} by a dimensionless function $f(r_i)$ of the dimensionless ratios, Eq. (11), the mass ratio M_A/M_P can be identically written as $M_A/M_P = r_0 f(r_i)$. [Note also that the d_{r_i}'s entering Eq. (14) above correspond to the differences $d_{r_i} - d_g$ in [5, 6] (with $d_g \equiv d_{r_0}$), because we defined above the ratios r_i by Eq. (11) which involved a logarithmic derivative of M_A/Λ_{QCD}, while we were working there with logarithmic derivatives of M_A/M_P.]

When contemplating, as we do here, possible variations over cosmological distances, the field φ must be essentially massless. However, in the present work we shall not need to consider any specific model neither for the mass (or self-potential $V(\varphi)$) of φ, nor for its matter couplings d_{r_0}, d_{r_i}. Indeed, the crucial point is that the observable acceleration (9) only depends on the effective charges (10), and on the various spatial gradient parameters $\nabla \ln r_i = B_{r_i}\,\hat{\mathbf{z}}_{r_i}$. This makes the present investigation quite model independent, as well as independent from the usual interpretation of local EP tests (which involve the bilinear products $(\alpha_A - \alpha_B)\,\alpha_E$).

[1] We have argued in [5, 6] that the composition dependence linked to the strange quark mass m_s was subdominant.

[2] Note that, from a theoretical point of view, the ratio $r_{\hat{m}} = \hat{m}/\Lambda_{QCD}$ is akin to $\mu = m_e/m_p$ in the sense that it is the ratio between a lepton mass and an hadronic one.

2. THE GRAVITATIONAL STARK EFFECT AND SPATIAL VARYING COUPLINGS

The accurate monitoring of the lunar motion (most notably by LLR experiments [9]) has led to impressive tests of relativistic gravity, and notably of various aspects of the EP [11]. Here we are interested in EP-violating effects in LLR that are linked to a *fixed* preferred direction in space. Such effects have been studied by Damour and Schaefer [12] in the context of binary pulsars. The analysis of such preferred-direction forces in the context of LLR has been done at leading order (LO) by Nordtvedt [13] (see also [14]) , and to very high perturbative order by Damour and Vokrouhlický [15] (using the Hill-Brown lunar theory [16]). For references to analytic studies of relativistic effects in lunar motion, as well as a self-contained introduction to Hill-Brown theory, see, e.g., [17]. Lunar dynamics is a notoriously difficult problem because of the rather strong perturbation coming from the Sun's tidal forces, which leads to badly convergent perturbation series in powers of the parameter $m = n'/(n - n') \simeq 1/12.3687$. [Here, $n' = 2\pi/(1\text{ yr})$ denotes the mean sidereal angular velocity of the Earth around the Sun, and $n = 2\pi/(27.32\text{ days})$ [18] the mean sidereal angular velocity of the Moon around the Earth.] For some effects, a LO perturbation treatment in m can be significantly inaccurate both because of the occurrence of small denominators, and of the slow convergence of the m-perturbation series. This is, for instance, the case for the Laplace-Nordtvedt effect of polarization of the Moon's orbit by an EP-violation linked to the Sun's gravity where higher-order terms in m increase the leading-order result by more than 62% [17, 19]. In the case of interest here of what has been called the "gravitational Stark effect" [12], i.e. the perturbing influence of a differential force (with a *fixed direction*) acting on a gravitationally bound two-body system, the situation is similar, though with significant differences.

Let us first recall that the classical (electric or gravitational) Stark effect is an example of singular perturbation where a small perturbing force can have a large effect. If we were approximating the dynamics of the relative Earth-Moon coordinate $\mathbf{r} = \mathbf{x_M} - \mathbf{x_E}$ in the presence of an external acceleration $\boldsymbol{\Delta}\mathbf{g} = \mathbf{g}_M - \mathbf{g}_E$ by means of the Lagrangian [after factorization of the Earth-Moon reduced mass $\mu \equiv m_E\, m_M/(m_E + m_M)$]

$$L = \frac{1}{2}\left(\frac{d\boldsymbol{r}}{dt}\right)^2 + \frac{G(m_E + m_M)}{r} + \boldsymbol{\Delta g}\cdot\boldsymbol{r}\,, \tag{15}$$

we could find the exact solution of the perturbed dynamics by separating the Hamilton-Jacobi equation corresponding to the Lagrangian (15) in parabolic coordinates ($\xi = r + z$, $\eta = r - z$, ϕ), with a z axis oriented along $\boldsymbol{\Delta g}$. One then finds that the exact solution corresponding to elliptic orbits undergoes a complicated secular evolution during which the osculating elements of the elliptic motion wander very far away from any given initial state. For instance, even if the perturbing acceleration $\boldsymbol{\Delta g}$ is very small, the osculating eccentricity will not undergo small oscillations around its initial value e_0 but will, on time scales $na/\Delta g$, take values quite different from e_0. This instability of elliptic motion under a constant force can also be seen by using the *averaged* evolution equations of the osculating orbital elements. More precisely, if we consider the evolution of the semi-major axis a, of the Lagrange-Laplace-Runge-Lenz eccentricity vector $\mathbf{e} = e\mathbf{a}$ (directed towards the periastron), where ($\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$) are orthornormal unit vectors with $\mathbf{a}$ pointing towards the periastron and $\mathbf{c}$ along the orbital angular momentum $\boldsymbol{\ell} = (1 - e^2)^{1/2}\mathbf{c} = \mathbf{r}\times\mathbf{v}$, one finds averaged evolution equations of the form [12]

$$\begin{aligned}\langle\frac{da}{dt}\rangle &= 0\ ,\\ \langle\frac{d\mathbf{e}}{dt}\rangle &= \mathbf{f}\times\boldsymbol{\ell}\ ,\\ \langle\frac{d\boldsymbol{\ell}}{dt}\rangle &= \mathbf{f}\times\mathbf{e}\end{aligned} \tag{16}$$

where

$$\mathbf{f} = \frac{3}{2}\frac{\Delta\mathbf{g}}{na}\ . \tag{17}$$

with n denoting the sidereal angular frequency of the Moon.

We see that while a stays secularly constant, the vectors $\boldsymbol{e}$ and $\boldsymbol{\ell}$ rotate one into another. More precisely (with $\boldsymbol{f} = f\,\hat{\mathbf{z}}$), one easily sees that, while e_z and ℓ_z stay constant, the two complex combinations $\varepsilon_x \equiv e_x + i\,\ell_y$ and $\varepsilon_y \equiv e_y + i\,\ell_x$ rotate as

$$\varepsilon_x(t) = e^{+ift}\,\varepsilon_x(0)\,;\quad \varepsilon_y(t) = e^{-ift}\,\varepsilon_y(0)\,, \tag{18}$$

leaving constant $|\varepsilon_x|^2 = e_x^2 + \ell_y^2$ and $|\varepsilon_y|^2 = e_y^2 + \ell_x^2$ (consistently with $\boldsymbol{e}^2 + \boldsymbol{\ell}^2 = 1 = \text{const.}$).

This Stark instability is rooted in the well known degeneracy of the Coulomb problem, *i.e.* the fact that the radial ω_r and angular frequencies ω_ϕ happen to be exactly equal, $\omega_r = \omega_\phi = n$, for a $1/r$ interaction potential. As a consequence, any lifting of the Coulomb degeneracy by an additional interaction potential (causing ω_r to differ from ω_ϕ) will tame the Stark instability. Ref. [12] considered the case where this lifting was due to the general relativistic modifications of the $1/r$ Newtonian potential. In that case the only modification of the secular evolution equations Eqs. (16) is the appearance of an additional contribution $+\,\omega_p\,\mathbf{c}\times\mathbf{e}$ on the r.h.s. of the evolution equation of $\mathbf{e}$, where $\omega_p = \omega_\phi - \omega_r$ is the precession frequency of the binary system, due to relativistic effects.

As a first orientation towards understanding the Stark effect in the lunar motion, let us start by assuming that, as in the case studied in Ref. [12], it is enough to replace the second secular evolution equation in Eqs. (16) by

$$\langle\frac{d\mathbf{e}}{dt}\rangle = \mathbf{f}\times\boldsymbol{\ell} + \omega_p\,\mathbf{c}\times\mathbf{e} \tag{19}$$

where ω_p describes the precession of the orbit of the Moon, which occurs with a period of 8.85 years [18]. [Let us note in passing that the amplitude of the eccentricity evolves according to $\frac{de}{dt} = (1-e^2)^{1/2}\mathbf{f}\cdot\mathbf{b}$.]

In addition, as the eccentricity of the Earth-Moon system is small $e = 0.0549$, we can, in first approximation (in view of the appearance of e on the r.h.s. of the evolution equation of the angular momentum $\boldsymbol{\ell}$), neglect the small wobbling of the direction of the orbital angular momentum $\mathbf{c}$ and approximate equation (19) (using also $\boldsymbol{\ell} = (1-e^2)^{1/2}\mathbf{c} \simeq \mathbf{c}$) by

$$\langle \frac{d\mathbf{e}}{dt} \rangle = \mathbf{f}\times\mathbf{c} + \omega_p\,\mathbf{c}\times\mathbf{e} = \omega_p\,\mathbf{c}\times(\mathbf{e}-\mathbf{e}_f) \tag{20}$$

where we have introduced

$$\mathbf{e}_f = \frac{\mathbf{f}_\perp}{\omega_p} = \frac{3}{2}\frac{\Delta\mathbf{g}_\perp}{na\omega_p} \tag{21}$$

where $\mathbf{f}_\perp = \mathbf{f} - (\mathbf{f}\cdot\mathbf{c})\,\mathbf{c}$ is the component of the external force in the plane of the orbit.

The general solution of Eq. (20) reads

$$\mathbf{e}(t) = \mathbf{e}_f + \mathbf{e}_p(t) \tag{22}$$

where the constant vector $\mathbf{e}_f$ describes a "forced eccentricity" (or a "polarization") induced by the external EP-violating force, and where $\mathbf{e}_p(t)$ is the usual, ω_p-precessing free eccentricity, which is allowed in absence of external Stark effect. Note in passing that the polarization of the elliptic orbit by the external force is oriented in the *opposite* direction of the projection $\mathbf{f}_\perp \propto \Delta\mathbf{g}_\perp$ of the force on the orbital plane. Indeed the eccentricity vector defines the direction towards the periastron, so that the elliptic orbit is mainly elongated in the apoastron direction $-\mathbf{e}_f \propto -\mathbf{f}_\perp$.

The final observable result, of this leading order treatment, linked to the polarization $\mathbf{e}_f$, is a sidereal frequency oscillation of the Earth-Moon range, connected with the direction of the projection $\mathbf{f}_\perp$ of the perturbing acceleration onto the orbital plane, of the form

$$\Delta^{LO} r(t) = \rho_f^{LO}\cos(n(t-t_0)-\phi_f) \quad . \tag{23}$$

where the (algebraic) amplitude is

$$\rho_f^{LO} = -\frac{f_\perp a}{\omega_p} = -\frac{3}{2}\frac{\Delta g_\perp}{n\omega_p} \tag{24}$$

and where $n(t-t_0)$ is the longitude of the Moon, and ϕ_f is the longitude of the direction $\mathbf{f}_\perp/|\mathbf{f}_\perp|$, both longitudes being measured within the orbital plane, from some common origin. [Neglecting the small inclination $i \simeq 5$ degrees of the Moon's orbit on the ecliptic, we can consider that both longitudes are ecliptic longitudes, counted from the vernal equinox.]

The above result was obtained as a leading-order approximation, under the simplifying assumption that the main effect of the solar tide on the Earth-Moon system was to introduce a perigee precession term in Eq. (19). However, the solar tide has more effects than this, and, as we recalled above, the theory of the lunar motion under the combined effect of the $G(m_E+m_M)/r$ Earth-Moon potential and of the quadrupolar tide $\frac{1}{2}x^i x^j \partial_{ij}(Gm_S/D)$ of the Sun, is a notoriously subtle theory, exhibiting many instances of slowly converging perturbation expansions in the (not so small) expansion parameter

$$m = \frac{n'}{n-n'} \simeq 0.080849 \simeq \frac{1}{12.3687} \tag{25}$$

Although the basic small parameter entering lunar theory is the ratio of the solar tidal potential to the Earth-Moon potential, which is of order $m^2 \sim 10^{-2}$, the lunar perturbation theory is not an expansion in powers of $m^2 \sim 10^{-2}$, but proceeds (beyond the LO term) in powers of $m \sim 1/12$ (see, e.g. [16]). We note, in particular, that the perigee precession frequency of the Moon is given by a perturbation expansion of the form

$$\frac{\omega_p}{n} = \frac{3}{2^2}\,m^2 + \frac{177}{2^5}\,m^3 + \frac{1659}{2^7}\,m^4 + \frac{85205}{2^{11}}\,m^5 + \frac{3073531}{2^{13}\cdot 3}\,m^6 + \cdots$$

which converges so slowly that the sum of higher-order terms approximately doubles the LO analytical result $\omega_p^{\rm LO}/n = 3m^2/4$ (see [20] for the literal computation of the perturbation expansion of ω_p/n up to the eleventh power of m). Let us also note that the above LO result for the range perturbation due to the Stark effect contained $\omega_p/n = 3m^2/4 + \cdots$ as a *small denominator* that significantly amplified the effect of the external perturbing force $\mathbf{f}$. [The presence of a small denominator is linked to the instability, recalled above, of elliptic motion under a constant force, because this small denominator tends to zero in the limiting case of the Lagrangian (15).]

The appearance of such small denominators oblige one to tackle in a more complete manner the Stark effect on the Moon's orbital motion. This was done by Damour and Vokrouhlický [15] using a Hill-Brown treatment (with the help of a dedicated algebraic computer programme). More specifically, Ref. [15] worked out the lunar range perturbation $\Delta r(t)$ induced by an external acceleration $\boldsymbol{\Delta}\mathbf{g}$ to very high order in the powers of m. This range perturbation $\Delta r(t)$ is the sum of many different frequency components that come from the nonlinear combination of the basic sidereal frequency n of the Moon (linked to the angular distance between the Moon and the external fixed direction $\boldsymbol{\Delta}\mathbf{g}$, or rather its projection $\boldsymbol{\Delta}\mathbf{g}_\perp$ on the Moon's orbital plane), with even multiples of the synodic frequency $n-n'$ linked to the angular distance (seen from the Earth) between the Moon and the Sun. Among the spectrum of combined frequencies $\pm(n+2j(n-n'))$ ($j \in Z$), two of them were found to be dominant: the basic sidereal frequency $\pm n$ (of period 27.32 days), and the $j=-1$ combination $\pm(n-2(n-n')) = \mp(n-2n')$ (of period 32.13 days).

The result of [15] can be written[3] as

$$\Delta\, r(t) = \rho_f \Bigg[\cos\left[n(t-t_0) - \phi_f\right] + \frac{15}{8}\, m\, \frac{S'_f(m)}{S_f(m)} \cos\left[n(t-t_0) - 2\tau(t) - \phi_f\right] + \ldots \Bigg] \tag{26}$$

Here $\tau(t) \equiv (n-n')t + \tau_0$ denotes the synodic phase, i.e. the angular distance between the Moon and the Sun, and the overall amplitude is given by

$$\rho_f = -2\, \frac{\Delta g_\perp}{n'^2}\, S_f(m) \tag{27}$$

where $\Delta g_\perp$ and ϕ_f are the magnitude, and the longitude, of the projection $\mathbf{\Delta g}_\perp$ of the external acceleration onto the lunar orbital plane[4], and where $S_f(m)$ and $S'_f(m)$ are m-perturbation series that start as $1 + O(m)$. For instance, the beginning of the expansion of $S_f(m)$ reads

$$S_f(m) = 1 - \tfrac{75}{8}\, m + \tfrac{235}{4}\, m^2 - \tfrac{127\,637}{384}\, m^3 + \tfrac{4\,172\,299}{2304}\, m^4 + O(m^5) \tag{28}$$

and Table IV of [15] gives the coefficients of this expansion to the ninth order in m. Even with such a high-order expansion one finds that the last term is still of fractional order 10^{-3}. This slow convergence is related to the presence of a pole in the series $S_f(m)$ and $S'_f(m)$ near $m_{\rm cr} \simeq -0.18407$. To get an accurate numerical estimate of these series Ref. [15] used a Padé resummation, with the results [for $m = m_{\rm Moon}$ given by Eq. (25)]

$$S_f(m) \simeq 0.5050$$

and

$$\frac{15}{8}\, m\, \frac{S'_f(m)}{S_f(m)} \simeq \frac{1}{5.94}$$

for the fractional coefficient of the subleading term with frequency $n - 2(n-n') = -(n-2n')$.

An observationally important aspect of the result (26) is the appearance of a specific combination of two harmonics, with known periods and phase, and with nearly comparable magnitudes. In particular, the fact that the amplitude of the $n - 2n'$ harmonic is only $\simeq 6$ times smaller than the LO n harmonic is a result of the subtleties of lunar perturbation theory. This term comes from the basic solar tide perturbation which is proportional to m^2, but it has been amplified to the $O(m)$ level by a small denominator (with the additional factor $15/8 \simeq 2$, leading to $15m/8 \simeq 1/6$). We have checked the presence of this subleading term by directly solving the *forced* Hill's equation [16, 17, 20] $d^2q(\tau)/d\tau^2 + \Theta(\tau)q(\tau) = \sigma(\tau)$ for the transverse perturbation $q(\tau)$ to Hill's variational orbit. Here, $\tau = (n-n')t + \tau_0$ as above. In that formulation, the frequency component $q_j \exp[i(1+m+2j)\tau]$ comes with the denominator $\theta_0 - (1+m+2j)^2$ which is small when $j=-1$, being $O(m)$ [namely, $\theta_0 - (m-1)^2 = [1 + 2m + O(m^2)] - (m-1)^2 = 4m + O(m^2)$].

For what concerns the leading term, with frequency n, in Eq. (26), it corresponds to the result Eq. (23) of the approximate treatment explained above. In both cases ϕ_f is the longitude of the external acceleration projected within the orbital plane. Note that if one were using the LO analytical results $(\omega_p/n)^{LO} = (3/4)m^2$ and $S_f^{LO}(m) = 1$, Eq. (24) would read $-2\Delta g_\perp/n'^2$ and would agree with Eq. (27). However, the exact result Eq. (27) is smaller than this by about a factor two, because of the correcting factor $S_f(m) \simeq 0.5050$. As noted in Ref. [15], when using in Eq. (24) the actual perigee precession ω_p (which is about twice larger than its LO estimate $(\omega_p/n)^{LO} = (3/4)m^2$), one captures most of the effect of the slowly converging series $S_f(m)$.

We can finally apply the result Eq. (26) to the EP-violating acceleration $\mathbf{\Delta g} = \mathbf{g}_M - \mathbf{g}_E$ where each $\mathbf{g}_A$ is given by equation (9). Let us first note that the result only depends on the amplitude and longitude of the projection on the orbital plane of the vectorial sum of the external accelerations

$$\mathbf{\Delta g} = \mathbf{g}_M - \mathbf{g}_E = -\sum_i (Q_{r_i}(M) - Q_{r_i}(E))\; B_{r_i} c^2\; \hat{\mathbf{z}}_{r_i}. \tag{29}$$

Alternatively, one could write the range perturbation as a sum of terms of the type of the r.h.s. of Eq. (26), each one having an amplitude $\rho_f(r_i)$ and a phase $\phi_f(r_i)$. [Note that we consider here algebraic amplitudes (that can be negative), and that the longitudes $\phi_f(r_i)$ always refer to the direction of the projection $\hat{\mathbf{z}}_{r_i\,\perp}$ of the gradient direction $+\hat{\mathbf{z}}_{r_i}$.] Let us, as it simplifies the writing of our results, make the natural assumption that all the cosmological gradients of the coupling constants are parallel to each other, i.e. $\hat{\mathbf{z}}_{r_i} = \hat{\mathbf{z}}$ independently of the label r_i, and let us denote by ϕ_f the longitude of the projected gradient direction $\hat{\mathbf{z}}_\perp$. This leads to a total range perturbation of the form Eq. (26), with a total (algebraic) amplitude of the form (after cancellation of two minus signs)

$$\rho_f^{tot} = 2S_f(m) \sum_i (Q_{r_i}(M) - Q_{r_i}(E))\; \frac{B_{r_i\perp} c^2}{n'^2} \tag{30}$$

where $B_{r_i\perp}$ is the magnitude of the projected gradient $B_{r_i}\hat{z}_\perp$.

Note that the numerical prefactor $2S_f(m) \simeq 1.010$ is close to 1, and that the parameter combination $B_{r_i\perp}c^2/n'^2$, where we recall that $n' = 2\pi/1\mathrm{yr}$, is the product of an acceleration by the square of a time, and

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[3] Here we change the notation of Section IV of [15] : e.g. $A_G \mathbf{N_G} \to \mathbf{\Delta g}_\perp$, $S_{\rm gal}(m) \to S_f(m)$, etc.

[4] Even in the full Hill-Brown treatment (with neglect of the lunar inclination, and of the eccentricity of the Moon's orbit), one finds that only the projection $\mathbf{\Delta g}_\perp$ of the external acceleration matters for the range perturbation.

is indeed a length. [Alternatively, we can think of it as the product of the spatial gradient $B_{r_i\perp} = [\text{length}]^{-1}$, by the squared length $c^2/n'^2 = (1\,\text{lyr})^2/4\pi^2$]. As a first orientation, note that a gradient of order $B \sim 10^{-6}\,\text{Glyr}^{-1}$, i.e. $Bc^2 \sim 10^{-12}\,\text{cm/s}^2$ (similar to the recent suggestions, Eqs. (2), (3), (12), (13)) corresponds to a figure of merit $Bc^2/n'^2 \simeq 25$ cm. In order to estimate the corresponding signal in lunar laser ranging, we need next to estimate the numerical value of the various EP-violating charge differences $Q_{r_i}(M) - Q_{r_i}(E)$ corresponding to the difference in composition of the Moon and the Earth. This will be the focus of the next Section.

Before tackling this issue, let us briefly mention that the central values of the equatorial coordinates $\alpha = 261$ degrees, $\delta = -58$ degrees of the cosmological gradient of the fine structure constant reported in [1] corresponds to an ecliptic latitude equal to $\beta = -34.7$ degrees, and an ecliptic longitude equal to $\lambda = -95.8$ degrees. The latter ecliptic longitude predicts[5] the value of the longitude entering the range perturbation Eq. (26), namely $\phi_f = \lambda$. On the other hand, the ecliptic latitude β enters the observable range ρ_f through the projection of the cosmological gradient direction onto the orbital plane, i.e.(essentially) onto the ecliptic. More precisely we have $B_{r_i\perp} = B_{r_i}\cos\beta$. Note that $\cos\beta = 0.822$ for the gradient reported by Webb et al. so that this projection (that we shall include in the estimates of the next Section) reduces only by 18% the full possible observable effect of such a gradient on the lunar motion.

3. EP VIOLATING CHARGES

Let us now estimate the numerical values of the various EP-violating charge differences $Q_{r_i}(M) - Q_{r_i}(E)$ entering the magnitude of the cosmologically induced Earth-Moon differential acceleration (9). This issue has been discussed in our previous work [5, 6] where we described the leading dependence of atomic masses on the parameters of the Standard Model. Because of the large neutron and proton masses, the dominant determining factor for all atomic masses is simply the scale of the strong interactions Λ_{QCD}. However, as already mentioned above, the main dependence of atomic masses on Λ_{QCD} is universal, and is conveniently factored out by rewriting each atomic mass M_A as $M_A = \Lambda_{QCD} f(r_i)$, where the r_i's (for $i \neq 0$) denote the dimensionless ratios Eq. (11). As a consequence, the charge Q_{r_0} associated to $r_0 = \Lambda_{QCD}/M_P$ via the general definition Eq. (10) is simply $Q_{r_0}(A) \equiv 1$, and the only non-zero contributions to the differential acceleration (9) will come from the dependence of M_A/Λ_{QCD} on the masses of the light quarks, on the electron mass and on the electromagnetic fine structure constant. For the quark masses, instead of considering separately the masses of the up and down quarks, it is convenient to consider their average and difference, namely

$$\hat{m} = (m_d + m_u)/2, \qquad \delta m = (m_d - m_u) \tag{31}$$

and to work with the fine structure constant α together with the dimensionless ratios

$$r_{\hat{m}} = \frac{\hat{m}}{\Lambda_{QCD}}, \quad r_{\delta m} = \frac{\delta m}{\Lambda_{QCD}}, \quad r_{m_e} = \frac{m_e}{\Lambda_{QCD}} \quad . \tag{32}$$

Because the fermion masses are the product of a dimensionless Yukawa coupling Γ_i and the Higgs vacuum expectation value (vev) v, $m_i = \Gamma_i v/\sqrt{2}$, these ratios can be considered as the product of the dimensionless Γ_i by the ratio v/Λ_{QCD} between the basic weak-interaction scale v and the basic strong-interaction scale Λ_{QCD}. In view of the independence of the mechanisms leading to the appearance of the two basic scales v and Λ_{QCD}, it seems a priori theoretically natural to expect that the cosmological gradients (if any) of the three mass ratios (32) will be of similar magnitudes, and therefore similar to that of the ratio $\mu = m_e/m_p$ for which the value (3) has been recently suggested.

Refs. [5, 6] derived the folllowing approximate estimates for the four effective charges Q_{r_i} associated to the three mass ratios (32), and to α:

$$Q_{\hat{m}} = F_A \left[0.093 - \frac{0.036}{A^{1/3}} - 0.020 \frac{(A-2Z)^2}{A^2} - 1.4 \times 10^{-4} \frac{Z(Z-1)}{A^{4/3}} \right], \tag{33}$$

$$Q_{\delta m} = F_A \left[0.0017 \, \frac{A-2Z}{A} \right], \tag{34}$$

$$Q_{m_e} = F_A \left[5.5 \times 10^{-4} \, \frac{Z}{A} \right], \tag{35}$$

and

$$Q_\alpha = F_A \left[-1.4 + 8.2 \frac{Z}{A} + 7.7 \frac{Z(Z-1)}{A^{4/3}} \right] \times 10^{-4}. \tag{36}$$

where $F_A \equiv A m_{\text{amu}}/M_A$, with $m_{\text{amu}} = 931$ MeV denoting the atomic mass unit. The common factor F_A is very close to 1, and we shall replace it by 1 in our estimates below.

Approximating the Moon as made of silicate (i.e. essentially SiO_2), and the Earth as made of a mantle of silicate and a core of iron (representing 32% of its mass), the above formulas yield the following Moon-Earth charge differences

[5] As above, we are here neglecting the small inclination of the Moon's orbit on the ecliptic. Note that taking into account this inclination, and its secular variation, would predict an additional small adiabatic variation of the amplitude ρ_f (and phase ϕ_f) of the range perturbation $\Delta r(t)$, corresponding to the secular wobbling of the lunar orbital plane.

$$Q_{\hat{m}}(M) - Q_{\hat{m}}(E) = -1.1 \times 10^{-3} \quad (37)$$
$$Q_{\delta m}(M) - Q_{\delta m}(E) = -3.8 \times 10^{-5} \quad (38)$$
$$Q_{m_e}(M) - Q_{m_e}(E) = +6.1 \times 10^{-6} \quad (39)$$
$$Q_{\alpha}(M) - Q_{\alpha}(E) = -3.1 \times 10^{-4} \quad (40)$$

We see that the most important charges for EP violation are $Q_{\hat{m}}$ and Q_α. This dominance of $Q_{\hat{m}}$ and Q_α over the other charges is true for most values of Z, A. It is related both to the rather small coefficients entering $Q_{\delta m}$, Eq. (34), and Q_{m_e}, Eq. (35), and to the fact that $A \simeq 2Z$ along the periodic table. If we use the facts that $F_A - 1 = O(10^{-3})$, and that $A \simeq 2Z$, we can simplify the composition dependence of the main EP-violating charges $Q_{\hat{m}}$ and Q_α as $Q_{\hat{m}} \simeq Q'_{\hat{m}}$+cst. and $Q_\alpha \simeq Q'_\alpha$+cst., where [5, 6]

$$Q'_{\hat{m}} = -\frac{0.036}{A^{1/3}} - 1.4 \times 10^{-4}\, \frac{Z(Z-1)}{A^{4/3}} \quad (41)$$
$$Q'_\alpha = +7.7 \frac{Z(Z-1)}{A^{4/3}} \times 10^{-4} \quad (42)$$

We list the values of $-Q'_{\hat{m}}$ and Q'_α for a sample of materials in Table I. This list shows that the maximum value of a charge difference would be the $Q_{\hat{m}}$ difference between a heavy element and a light one with $Q_{\hat{m}}(\text{heavy}) - Q_{\hat{m}}(\text{light}) \simeq +10^{-2}$. The corresponding maximum acceleration level $\Delta g_{\hat{m}}^{\max} = \Delta Q_{\hat{m}}^{\max} B_{\hat{m}} c^2 \simeq 10^{-2} B_{\hat{m}} c^2$ would numerically be

$$\Delta g_{\hat{m}}^{\max} \simeq \left(\frac{B_{\hat{m}}}{10^{-6}\ \text{Glyr}^{-1}} \right) \times 10^{-14} \text{cm/s}^2$$

for a cosmic gradient of order of those reported by Webb et al.

TABLE I: Approximate EP-violating 'effective charges' for a sample of materials. These charges are averaged over the (isotopic or chemical, for SiO_2) composition.

Material	A	Z	$-Q'_{\hat{m}}$	Q'_α
Li	7	3	18.88 $\times 10^{-3}$	0.345 $\times 10^{-3}$
Be	9	4	17.40 $\times 10^{-3}$	0.494 $\times 10^{-3}$
Al	27	13	12.27 $\times 10^{-3}$	1.48 $\times 10^{-3}$
Si	28.1	14	12.1 $\times 10^{-3}$	1.64 $\times 10^{-3}$
SiO_2	...	...	13.39 $\times 10^{-3}$	1.34 $\times 10^{-3}$
Ti	47.9	22	10.28 $\times 10^{-3}$	2.04 $\times 10^{-3}$
Fe	56	26	9.83 $\times 10^{-3}$	2.34 $\times 10^{-3}$
Cu	63.6	29	9.47 $\times 10^{-3}$	2.46 $\times 10^{-3}$
Cs	133	55	7.67 $\times 10^{-3}$	3.37 $\times 10^{-3}$
Pt	195.1	78	6.95 $\times 10^{-3}$	4.09 $\times 10^{-3}$

4. OBSERVATIONAL SIGNALS LINKED TO POSSIBLE COSMOLOGICAL GRADIENTS

Putting together our results, keeping only the dominant terms linked to $Q_{\hat{m}}$ and Q_α, and scaling the possible cosmological gradients of $\hat{m}$ and α by the recently reported values[6] Eqs. (3), (2) [and (13), (12)], we conclude that those cosmological gradients entail EP-violating differential accelerations on the Moon directed along the unit vector $\hat{\mathbf{z}}_\perp$ (with ecliptic longitude equal to $\phi_f \simeq \lambda \simeq -95.8$ degrees), and with algebraic magnitudes

$$\Delta g_\perp = \Delta g_{\hat{m}\perp} + \Delta g_{\alpha\perp} \quad (43)$$

where

$$\Delta g_{\hat{m}\perp} = -(Q_{\hat{m}}(M) - Q_{\hat{m}}(E))\ B_{\hat{m}\perp} c^2 \quad (44)$$
$$= +2.2 \left(\frac{B_{\hat{m}}}{2.6 \times 10^{-6}\ \text{Glyr}^{-1}} \right) \times 10^{-15} \text{cm/s}^2$$

and

$$\Delta g_{\alpha\perp} = -(Q_\alpha(M) - Q_\alpha(E))\ B_{\alpha\perp} c^2 \quad (45)$$
$$= +2.7 \left(\frac{B_\alpha}{1.1 \times 10^{-6}\ \text{Glyr}^{-1}} \right) \times 10^{-16} \text{cm/s}^2$$

These differential accelerations then induce a perturbation in the Earth-Moon range which has the specific time signature (26) with an algebraic magnitude

$$\rho_f = \rho_{\hat{m}} + \rho_\alpha \quad (46)$$

where

$$\rho_{\hat{m}} = -\left(\frac{B_{\hat{m}}}{2.6 \times 10^{-6}\ \text{Glyr}^{-1}} \right) 0.59\,\text{mm} \quad (47)$$

and

$$\rho_\alpha = -\left(\frac{B_\alpha}{1.1 \times 10^{-6}\ \text{Glyr}^{-1}} \right) 0.068\,\text{mm} \quad (48)$$

As we see, the dominant effect is expected to be linked to the cosmological gradient of $\hat{m}/\Lambda_{QCD}$, so that the recent findings of Webb and collaborators [1, 2] suggest the presence of millimeter-level sidereal fluctuations in the Earth-Moon range. Such fluctuations, if they exist, should be detectable by the APOLLO experiment. Indeed, this experiment has shown its capability of obtaining "normal-point" range measurements with nightly median uncertainty of 1.8 mm for their entire data set, and 1.1 mm for their recent data [8]. By accumulating millimeter-level range data over a sufficiently long time (comparable to the perigee period $\simeq 8.85$ yr), the APOLLO experiment should be able both to decorrelate the specific sidereal signal (26) from the many existing Newtonian range effects (which include synodic, $n - n'$, and anomalistic, $\omega_r = n - \omega_p$, frequencies), and to measure its amplitude ρ_f to a fraction of a millimeter. Depending on the result of such an analysis, the

[6] As mentioned above, it is natural to expect that a cosmological gradient of $\mu = m_e/m_p$ implies a similar gradient in the weak-interaction/strong-interaction ratios r_i, and notably in $\hat{m}/\Lambda_{QCD}$.

APOLLO experiment could either establish the reality of a cosmological gradient of coupling constants, or set upper bounds on the gradients $B_{\hat{m}}$ and B_α (or more precisely on the combination $B_{\hat{m}\perp} + 0.28 B_{\alpha\perp}$ entering $\Delta g_\perp$) at levels smaller than the levels Eqs. (3), (2) [and (13), (12)].

Let us mention in this respect that a sidereal range perturbation of the approximate form (23), with a phase ϕ_f linked to the center of the Galaxy, has been searched for in the pre-APOLLO, few-centimeter-level LLR data after the suggestion of Ref. [13]. Nordtvedt, Müller and Soffel [21] published an upper limit of

$$\Delta g_{\rm gal} < 3 \times 10^{-14} {\rm cm/s}^2 \tag{49}$$

on a possible perturbing differential acceleration linked to the Galactic center, while a further analysis of Müller (cited in [22]) obtained

$$\Delta g_{\rm gal} = (4 \pm 4) \times 10^{-14} {\rm cm/s}^2 \tag{50}$$

The gain in sensitivity of the APOLLO experiment, by more than a factor 10, and the richer time signature of the more complete signal (26), make us expect that it will be possible to probe the acceleration levels (44), (45), above.

It is instructive to compare the (potential) sensitivity of LLR experiments to external EP-violating accelerations with the sensitivity of other EP experiments. There have been constraints on anomalous cosmic accelerations by laboratory based EP tests. The most precise is quoted as a differential acceleration in any direction of the sky [10]

$$|g({\rm Be}) - g({\rm Ti})| < 8.8 \times 10^{-13} \ {\rm cm/s}^2 \ (95\% \, C.L.)$$

The charge differences $Q_{r_i}({\rm Be}) - Q_{r_i}({\rm Ti})$ are again dominated by $Q_{\hat{m}}({\rm Be}) - Q_{\hat{m}}({\rm Ti}) = -7.23 \times 10^{-3}$ and $Q_\alpha({\rm Be}) - Q_\alpha({\rm Ti}) = -1.56 \times 10^{-3}$. Assuming, as above, that the effect of the gradient of $\hat{m}$ (which couples to the dominant charge difference) dominates, the above upper bound on $|g({\rm Be}) - g({\rm Ti})|$ can be readily converted to a bound on the corresponding cosmological gradient $B_{\hat{m}}$, namely

$$|B_{\hat{m}}| < 1.3 \times 10^{-4} \ {\rm Glyr}^{-1} \tag{51}$$

This is is weaker than the recently suggested (theoretically similar) gradient Eq. (3) by a factor $\simeq 50$. Such a difference in *acceleration sensitivity* between Earth-based EP experiments and LLR ones might seem surprising in view of the fact that both types of experiments currently lead to comparable limits on the (Eötvös) EP-violation parameter $\eta = \Delta g/g$, namely $\eta_{\rm EarthMoon} = (-1.0 \pm 1.4) \times 10^{-13}$ [9], versus $\eta_{\rm BeTi} = (0.3 \pm 1.8) \times 10^{-13}$ [10], and that both types of experiments use comparable background accelerations g in the ratio $\Delta g/g$. [Indeed, the g due to the Sun at Earth is $g_S \simeq 0.6$ cm/s^2, while torsion balance experiments use only the horizontal component of the Earth gravity, namely $g_{E\perp} \simeq 1.7$ cm/s^2 at a latitude of 45 degrees.] We note that the greater sensitivity of LLR experiments to external (especially fixed-direction) accelerations is essentially rooted in the specific Stark instability mentioned above. Indeed, generally speaking, a differential acceleration Δg acting during a characteristic time t_c (which is $t_c \sim \omega^{-1} = T/(2\pi)$ for a periodic phenomenon of angular frequency ω and period T) corresponds to a measurable displacement of order $\Delta r \sim \Delta g t_c^2 = \Delta g/\omega^2$. In the LLR case, we saw above that the range perturbation is $\Delta r \sim \Delta g/n'^2$ which is *larger* than the expected perturbation $\sim \Delta g/n^2$ associated to the lunar frequency n by a factor

$$\left(\frac{n}{n'}\right)^2 = \left(\frac{1 {\rm year}}{1 {\rm sidereal\,month}}\right)^2 = (13.37)^2 = 178.7$$

This amplification factor lies at the root of the increased sensitivity of LLR experiments to external EP-violating accelerations having a fixed direction. We note in passing that the LLR sensitivity to the usually considered Laplace-Nordtvedt solar-rooted EP-violating acceleration is only amplified, w.r.t. $\Delta g/n^2$, by a parametrically smaller factor $\sim (3/2)(n/n') \sim 20$, i.e. about ten times less than in the "Stark", fixed direction case. This difference is due to the difference in the corresponding nearly resonant denominators, namely, in Hill's equation, a denominator $\theta_0 - (1+m)^2 = [1 + 2m + O(m^2)] - (1+m)^2 = O(m^2)$ in the sidereal-frequency (Stark) case, versus $\theta_0 - (1)^2 = [1 + 2m + O(m^2)] - 1 = 2m + O(m^2)$ in the synodic-frequency (Laplace-Nordtvedt) one.

Let us finally note that presently planned improved EP tests such as the Satellite Test of the Equivalence Principle (STEP) [23] ($\eta \sim 10^{-18}$) or proposed cold-atom-technology tests (($\eta \sim 10^{-17}$) [24], which will make use of the full strength of the Earth gravity, $g_E \simeq 980\,{\rm cm/s}^2$, will be able to probe the cosmological-gradient-induced differential accelerations discussed above. Indeed, the acceleration (44) linked to a cosmic gradient of $\hat{m}$ can be rewritten (modulo a cosine factor, with a specific time dependence linked to the projection onto the sensitive direction of the considered EP experiment) as

$$\eta = \frac{\Delta g}{g} = 2.5 \, \frac{g_E}{g} \left(\frac{\Delta Q_{\hat{m}}}{10^{-2}}\right) \left(\frac{B_{\hat{m}}}{2.6 \times 10^{-6} \ {\rm Glyr}^{-1}}\right) \times 10^{-17}$$

where we allowed the considered man-made EP test to optimize the choice of materials by having a $\Delta Q_{\hat{m}} \sim 10^{-2}$, i.e. ten times better than for the Earth-Moon case (see Table I). This result shows that the LLR test of the cosmological acceleration (44), that should be doable by the APOLLO experiment, corresponds (from the point of view of the sensitivity to a cosmic gradient) to an Earth-based EP test at the $\eta \sim 10^{-17}$ level.

Acknowledgements

We thank Eric Adelberger for useful information about the APOLLO experiment. JFD thanks the IHES for hospitality during this project and also acknowledges partial support by the NSF grants PHY - 0855119 and by the Foundational Questions Institute.

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