In memoriam: Cécile DeWitt-Morette

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I. The eventful life of Cécile

Let me start by briefly discussing Cécile's early life. Her father, an important industrialist, died early. Her mother remarried with the principal collaborator of her late husband. Cécile spent her early years at the Ecole des Mines de Paris[1], a beautiful building facing the Luxembourg garden! Then the family moved to Caen. During the war, Caen was heavily bombed in June 1944 by the US Air Force, without a compelling military excuse. The home of the parents of Cécile was destroyed, her mother and sister being killed. Fortunately, Cécile Morette was not there. After the complete invasion of France, in the fall 1942, by German and Italian troops, France was divided into 8 different zones and an "Ausweis" (= passport) was required for interzone travel. Cécile had been given the permission to travel between Caen and Paris to attend de Broglie's lectures: she was in Paris during the bombing!

After the war, she began working on a thesis under L. de Broglie. Soon after, she was hired by Frédéric Joliot(-Curie). Joliot, who, after surviving during the Nazi invasion of France, had become the head of France nuclear program (both civilian and military) as well as a fanatical communist militant, was too busy to prepare his lectures at Collège de France, and delegated the preparation to the rather inexperienced Cécile. Then she visited Great Britain and Ireland with the mission of interviewing important physicists about their work during the war. After a memorable encounter with Dirac, more talkative than customary, she visited the Institute for Advanced Study in Dublin. There she met E. Schrödinger, who had escaped from nazi Austria with his two wives and was welcomed by the Cardinal! There was also Heitler, who suggested to Cécile to work on meson physics, especially the distinction between π and μ meson. That was the subject of her thesis, defended in front of de Broglie, after her return in Paris.

At the invitation of Robert Oppenheimer Cécile visited the Institute for Advanced Study in Princeton. Under the benevolent leadership of Oppenheimer,

[1] If I remember correctly, her grandfather was the general secretary of the school.

she started a lifelong friendship with Freeman Dyson, a more complicated relationship with Richard Feynman[2] and met Bryce DeWitt, then a veteran of US Air Force during second world war. He became eventually her husband. That was also the beginning of Les Houches enterprise.

At the time, in the fifties, experimental physics (and chemistry) was still strong in France, mainly as part of the Curie heritage. Nuclear physics, magnetism (remember the Weiss' domains[3]), solid state physics (created by Pierre Aigrain and developing later into ultralow temperatures by Cohen-Tannoudji and his school). On the other hand, theoretical physics was poor. The school of de Broglie, filled with incompetent and sometimes dishonest people, went into a deadlock by refusing the "Copenhagen interpretation" and developing tools like "the double solution" going nowhere. Louis de Broglie considered that quantum mechanics was like a luxurious jewel, for the happy fews. As a result, we had to wait until the late sixties for the creation of regular graduate, (then undergraduate) courses about quantum mechanics. In the fifties, Feynman lectures, Landau and Lifschitz were not available, and I learned myself quantum mechanics from the giant books of Hermann Weyl and P.A.M. Dirac.

Gravitation and General Relativity were not in a better situation. The founding fathers in France, namely Georges Darmois, Marie-Antoinette Tonnelat, André Lichnerowicz were skilled mathematicians, who developed it like a purely mathematical theory. Yvonne Choquet-Bruhat came later with more interest in physics. I remember discussing around 1970 with Lichnerowicz the first experiments by Weber. He was not at all interested despite his own mathematical contributions to the existence of gravitational waves. It was the team of theoretical physicists at Meudon (Brandon Carter, Thibault Damour) who transformed, within the French community, General Relativity into a physical discipline[4].

To fill this vacuum, after marrying Brice DeWitt, Cécile came with a very original and bold idea, to create a summer school in physics, where, in the inspiring place of Les Houches in the Alps, students and professors would share life and science for two months. This was started in 1951 and brought teachers like Pauli or Feynman. The place has been a landmark in theoretical physics for 70 years.

For me, it was a "missed opportunity", which influenced my own carrier. The first sessions were attended by some of my classmates, like Pierre-Gilles de Gennes and Bernard Malgrange. I was then 22 and full of youthful dreams, ranging from theology (I had just given up the idea of becoming a minister in the French Calvinist Church), to unorthodox Marxist philosophy (following my professor of philosophy Louis Althusser) and to radioastronomy (then a new science). But in between, I had been seduced by Bourbaki, and put all of my energy in their ambitious program of reconstructing mathematics from scratch.

[2]His relationship with women was always complicated.
[3]Henri Cartan was the son in law of Weiss!
[4]It took fifty years to realize the dream of observing these gravitational waves!

After Princeton and the marriage of Cécile and Bryce, came the period of Chapel Hill. For 16 years she was unable to get a position there, because of a rule forbidding a married couple to be hired in the same department. She was the mother of four daughters, as efficient as a mother as in any of her enterprises. One of her daughters, married in an orthodox jewish family, and is a writer. Another one is an important figure of the World Bank. Still another one is mentally handicaped by obsessional troubles, and needs a daily help. She was a big concern for Cécile, but when you are Cécile, one answer is to become an important figure in the world of charitable foundations (Red Cross and others), to lobby for handicaped people among Washington senators, and to organize fundraising events, with John Nash as a guest star after his "recovery".

Then came the fortunate move from Chapell Hill to Austin. Austin is a very pleasant campus and downtown[5]. On both gown and town you have a wealth of theatrical and musical events. Besides the "President Johnson Library" and the football field, you have a big auditorium seating 3000 persons, where you can watch the Covent Garden, three nights in a row, for 20 USD a night.

Despite Texas climate, the town is green, with many parks, an artificial lake and a "natural" pool in a canyon, where you swim in the middle of turtles (and perhaps snakes!). To sum up, Austin, both campus and town are liberal within an harshly conservative Texan environment. A standard saying is: "If Texas secedes[6] some day from the United States, the day before Austin will secede from Texas".

Cécile was at first appointed to the Department of Astronomy. The dominant figure was there Gérard de Vaucouleurs[7], a French émigré, who together with his wife maintained and developed a catalogue of galaxies. Now that we estimate the number of galaxies to be of the order on 10^{11} (each galaxy with 10^{10} stars in the mean), I don't know the purpose and meaning of such a catalogue today. Many observations were done at the MacDonald Observatory, and people commuted between Austin and MacDonald (a roundtrip drive of 1500 kms in Texas desert). Nowadays, only people in charge of maintaining the instruments visit MacDonald, and astronomers watch the sky through their computer screen.

After some years, she joined the Physics Department. This was a strong department, chaired by S. Weinberg, with Y. Neeman and I. Prigogine as frequent visitors. Weinberg, a Nobel laureate, was a superb scientist, with a list

[5]Not everyone realizes that Austin is a dual city, the Mexican one on the eastside, and the "wasp" one on the westside, separated by the physical barrier of Interstate 35. Now that the airport has moved to the East side, visitors have more difficulties ignoring this geographical feature. Also, in the "Cinque de Mayo" (5 May, Mexican national festival), the town belongs to the Mexicans. Notice also that, in an academic building after 8 p.m., if you need a key, better ask about it in spanish!

[6]I doubt that after the civil war, Texas would be permitted to secede, but there is a bylaw allowing Texas to split into five pieces, each one sending two senators in Washington Capitol. A similar bylaw applies to California.

[7]The Vaucouleurs had a very gifted technician, who decided once to run with his daughter a (very successful) business of French bakeries. Before leaving, he had to swear to visit Austin for a week every year: he was the only one able to unlock some secret tools!

of beautiful books (including one in cosmology), but a strong character who claimed to know everything[8]. Despite political differences[9], I became a friend of Neeman and we shared students and seminars. On the other hand, Prigogine was disappointing. When he annonced a seminar talk, it was eventually delegated to one of his assistants. Nonequilibrium physics is a fascinating subject, but he used the wrong tools for that[10].

A component of the Physics Department was the Center for Relativity. It was created by J.A. Wheeler, after his retirement from Princeton University. Besides Wheeler was Bryce DeWitt, who had been interested in Quantum Gravity from the beginning. Eventually, Bryce became the director of this center and Cécile joined in !!. There was another character in this center, whom we called the "captain" because he was later on associated with joint experimental programs of the Navy and the NASA (first versions of the GPS system!).

Both Cécile and Bryce were adventurous characters. Before telling their adventure in Mauritania to watch a solar eclipse, I have to insert a comment on the experimental basis of Einstein's gravitation theory. Three tests of General Relativity have been historically important:

a) *The residual motion of Mercury's perihelion* ($43''$ of arc per century!). Despite many efforts, and many proposals (a new planet, Vulcan, between Sun and Mercury, an oblate Sun, with quadrupolar effect, modification of Newton's law) no astronomical explanation was successful, while Einstein's calculations in 1915 fitted very well.

b) *The redshift of spectral lines.* Stellar data were quite inconclusive, and only through the use of the Mossbauer effect on earth the experiment of Pound and Racka in 1959 confirmed this redshift, now a central element in astrophysics!

c) *The bending of light rays around the Sun.* Here there is a crucial factor of 2. According to a calculation going back to Laplace, and assuming that a light ray corresponds to the motion of a particle (photon?), gravitational influence by the Sun results in a Keplerian hyperbola, independent of the mass of the photon by Galileo's law of free fall. The deviation, that is the angle between the two asymptotic directions, is 2α radians, in the first order in $\alpha = \frac{GM_\odot}{bc^2}$ (G gravitation constant, c speed of light *in vacuo*, b impact parameter, $M_\odot$ mass of the Sun). Numerically $\alpha \sim 2.10^{-6}$ for a ray grazing Sun.

In 1907, Einstein tried for the first time to account for the influence of gravitation on light. He assumed that at a place $\boldsymbol{x}$ where the gravitation potential is Φ, the speed of light is $\sqrt{c^2 + 2\Phi}$ (a consequence of the conservation of energy for a particle!). To include relativity effects, we have to modify the Minkowski

[8]It took me time and flexibility to be respected by him. I was helped in this endeavor by a visit of Ludwig Faddeev, who treated me like an old friend!

[9]Neeman had many political and military responsabilities in Israël, being the boss of Mossad during the 1967 war, and the father of Israël nuclear weapons. In my opinion he would have deserved to be a Nobel laureate, but perhaps the Nobel committee was afraid of political controversies.

[10]I once advised a graduate student in his group to go elsewhere after being granted his Ph.D.

metric $\eta = c^2\,dt^2 - d\boldsymbol{x}^2$ by replacing c^2 by $c^2+2\Phi$. If we don't change the spatial metric $d\boldsymbol{x}^2$, we get in first approximation in α the same result 2α radians. In polar coordinates, we have $d\boldsymbol{x}^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta \cdot d\varphi^2)$. But taking in full account the field equations for gravitation (discovered by Einstein in November 1915!), we have to replace dr^2 by $dr^2/\left(1+2\frac{\Phi}{c^2}\right)$ with the gravitational potential $\Phi = GM_\odot/r$ (Schwarzschild). To calculate the deviation to first order in α, it is enough to use an approximate solution discovered by Einstein in 1915. If we take this into account, the deviation is $2\times 2\alpha$ radians (equal to $1,7''$ of arc). Here is the factor of 2. Eddington in 1919 claimed to have observed this.

Let us go back to Bryce and Cécile! They decided to improve on previous measurements of light deflection by taking advantage of a solar eclipse in 1974 visible in Mauritania. They carried heavy equipment, tried to insulate it from the ambiant heat, and flew over the desert. But Bryce was an experienced pilot from his war experience[11].

According to their measurements, they could guarantee an angular deviation of 4α radians, *up to an error of 40%. To conclude, there was really no observational basis for General Relativity before 1960.*

Things have changed: within the solar system the gravitational field is of order α at most (weak field) but the post newtonian approximation program, with data fitting of numerous space observations, gives an excellent agreement. For strong fields, binary pulsars and more recently, gravitational waves give a nonambiguous confirmation.

Before describing our collaboration, let me give a few examples of the managerial ability of Cécile.

Once in Les Houches, during one of my attendances there, there was no one to serve breakfast on a Sunday morning: the two local aids in the kitchen had been out dancing for the whole night. Ten minutes later, everything was in order with Cécile preparing herself coffee and toasts!

Much later, there were discussions about recruiting her in the board of trustees of IHES, and some people objected about her age, being afraid that she would sleep during the meetings. I told everyone that no one can sleep in a meeting where she is present! Indeed, she usually came a few days in advance, meeting everyone in the staff to know everything by herself. One of her last scientific works was a publication of Bryce's papers[12].

Finally, she had a quite sizable estate in France, especially a family house in Pornique, on the Ocean, and a farm in Amélie-les-Bains near the Spanish border. Bryce would visit it every summer, cutting trees like a modern Titan! Before dying, she managed a nonobjectionable transfer to her daughters!

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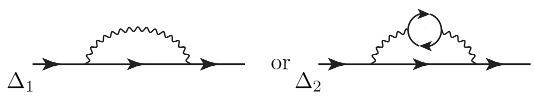
[11] In the beginning of the space exploration, he volunteered and went through the physical tests, only to be turned down because he was over forty!

[12] *A pursuit of quantum gravity. Memoirs of Bryce DeWitt*, Springer, 2000.

II. My collaboration with Cécile

It lasted more than 20 years and ended with the publication of a book[13]. It started in two distinct ways. When Cécile visited Princeton in the 1950's the last news in physics were the development of *Quantum Electrodynamics*. What was needed was a mathematical model to account for the newly observed Lamb shift, at a deeper level account for the radiative corrections to atomic spectra. In classical terms, the reaction on an electron of mass m of the electromagnetic field emitted by this electron is infinite (for a pointlike electron). In quantum theory, the effect is still infinite, but with a logarithmic (and not linear) divergence. Bethe proposed a *cutoff*: the frequencies higher than a bound $\Lambda = \frac{mc^2}{h}$ should be ignored. The challenge was to make it compatible with (special) relativity. Using a very hazardous proposal ("path integrals" or "sum over histories"), R. Feynman gave a recipe using his well-known diagrams. For instance, for one incoming and one outgoing electron, intermediate stages can be pictured by diagrams like



(solid lines for electrons, wavy lines for photons). If the incoming (and outgoing) 4-momentum is $\boldsymbol{p}$, then each diagram Δ contributes an amplitude $A_\Delta(\boldsymbol{p})$ and one estimates probabilities by $\left|\sum_\Delta A_\Delta(\boldsymbol{p})\right|^2$. Feynman gave explicit formulas to calculate $A_\Delta(\boldsymbol{p})$ by means of integrals. The complexity of these integrals increases with the number of *loops* in the diagrams (1 for Δ_1, 2 for Δ_2, etc...). The result can be expanded into a power series in the coupling constant $\alpha_e = \frac{e^2}{4\pi\varepsilon_0}$. Each such integral diverges logarithmically, but putting a Bethe-type cutoff Λ makes them convergent, and the recipes of renormalization tell us how to go to the limit $\Lambda \to \infty$.

That was an immediate success, most notably by the calculation of the Lamb shift and anomalous magnetic moment of the electron. The intuitive character of the diagrams made them very popular, but Feynman's justification by path integrals seemed hazardous, and was ignored by most people. Soon after, F. Dyson rediscovered under the name of "time-ordered exponential series" a familiar tool in ordinary differential equations (Lappo-Danilevsky), and gave an *ab initro* derivation of Feynman's formulas.

Some years later, Feynman gave a basic course about path integrals, which eventually was written up by his student Hibbs and published under their two names. This was not a success, and the book remained very unconvincing. Why? I don't know. There were numerous attempts by both physicists and mathematicians, in the West as well as in Soviet Union, to formulate correctly

[13]P. Cartier and C. DeWitt-Morette, *Functional integration (Action and symmetries)*, Cambridge Univ. Press, 2006.

the path integrals. But before describing the proposal made by Cécile and myself in our book, let me describe the other development, running in parallel: *history of measure theory and Lebesgue integral.*[14]

Lebesgue integral was invented around 1900, and underwent many developments until 1940. A notable event was the proof by Markoff in 1938 of a vast generalization of previous results of J. Hadamard and F. Riesz. This result establishes a bijection, between regular Borel measures[15] μ on a compact space X, and the linear forms $\overline{\mu}$ on the Banach space $C^0(X;\mathbb{R})$ of continuous real-valued functions on X, which satisfy $\overline{\mu}(f) \geq 0$ for $f \geq 0$. The correspondence is given by $\overline{\mu}(f) = \int_X f \cdot d\mu$. As a bonus, the Lebesgue space $L^p(X,\mu)$ is the completion of the space $C^0(X;\mathbb{R})$ under the norm $N_p(f) = \left(\int_X |f|^p d\mu\right)^{1/p}$. This result is readily extended to a locally compact space, and A. Weil saw this duality approach as the right foundation for a general integration theory. His views were supported within Bourbaki by L. Schwartz, who liked the idea of duality and used it in the creation of the theory of distributions. R. Godement put all of his energy to pursue the project which ended by the publication of Bourbaki in several volumes around 1950.

The method worked well for *locally compact* spaces. But Bourbaki was obsessed by the idea of removing any countability assumption from topology and analysis – at the cost of rather akward definitions, like the one of measurable sets or functions (also the various definitions of the exterior measure of a set!). The limitation to locally compact spaces didn't look serious for geometrical and even number-theoretical applications.

As long as probability theory focused on finite (or even countable) families of random variables, one needed only integration over finite-dimensional manifolds (hence locally compact spaces) but the rapid development of random processus required new tools, and Bourbaki decided to complete his treatise on integration. This was done in the 1960's. Officially, every text in the Bourbaki treatise is anonymous, and it is really a team work. Today, I can say that the impetus came from L. Schwartz, P.A. Meyer and myself. Schwartz had as father-in-law, Paul Lévy, a famous probabilist, P.A. Meyer was then a star among French (and world) probabilists, and myself was motivated by my interest in probability and mathematical physics. After many strenuous efforts, it was generally accepted that the right category of spaces were the *Polish spaces* (separable, complete, metric) or some slight generalizations (Souslin or Lusin spaces). Borel measures were automatically regular and the Soviet School (Gelfand, Prokhorov, Minlos, ...) had established the existence of infinite products of measures, or more generally projective limits. There a compactness criterion, given by Prokhorov, gives the final answer.

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[14]A readable account is given by Bourbaki (see *"Eléments d'histoire des mathématiques"*, Hermann, Paris, 1960, 2[nd] édition, Springer Verlag, 2007).

[15]A Borel measure on a compact space X is a nonnegative σ-additive function μ on the σ-algebra of Borel subsets of X. Regularity means that for any measurable subset E of X and any $\varepsilon > 0$, there exist two subsets, K compact, U open such that $K \subset E \subset U$ and $\mu(U) - \mu(K) \leq \varepsilon$. Regularity is automatic if there is a countable basis of open sets in X.

The most innovative section in the new chapter of Bourbaki deals with *promeasures*, a linear version of the general method sometimes called cylindrical measures[16]. Following S. Bochner (1960) and generalizing a method invented by P. Lévy in the 1920's one introduces the *Fourier transform* of a promeasure in the linear space X, a function on the dual space X' of X. By Minlos theorem, if the space X is *nuclear* in the sense of Grothendieck's thesis, the continuity of the Fourier transform on X' is what is needed to have a true measure on X, instead of the approximating promeasure.

Here the two trends came together. It started in an exchange between J. Dieudonné and Cécile in 1971. She was mentioning to him that in physics one needs oscillating integrals like $\int_{-\infty}^{+\infty} e^{ix^2} dx$ instead of convergent integrals like $\int_{-\infty}^{+\infty} e^{-x^2} dx$. Dieudonné replied that Fourier transforms exist for distributions and take care of some of the oscillating integrals. His suggestion was to replace *pro-measures* useful in probability theory by *pro-distributions* and to ask Cartier about that. A week later, I met Cécile at a party at IHES, and so started our long-term collaboration and friendship. So, I could repair the "missed opportunity". At last my investment in writing the Bourbaki treatise could bring fruits for our collaboration.

Our scientific marriage lasted more than 20 years. Cécile had already published two books[17] in collaboration with Yvonne Choquet-Bruhat and the first idea was a book by her and her husband, together with mathematical appendices by myself. This could not be done for scientific and personal reasons, and soon we agreed on writing together a book[18], while Bryce DeWitt published his own version[19]. It required me to visit Austin every year, visits I enjoyed very much. I was the guest of both the Physics and Mathematics departments. On both sides, I had good connections, most notably John Tate on the Math. department. I shared graduate students with Cécile. I particularly remember a young Chinese Lady by the name of Xiao-Rong Wu, who needed less than a year to transform from a shy Chinese girl into a standard American girl student, marrying an American. Also, a German by the name of Saemann (whom I met recently in Edinburg) and Markus Berg a Swede who came back to his country because he was not a member of the "string fashion" in the USA; he offered a wonderful wedding party in Austin. Cécile and myself organized many scientific events in Les Houches, Cargese, Institut Henri Poincaré, Strasbourg, etc. ... My wife and I visited Texas, Louisiana, New-Mexico in memorable travels and as mentioned already, I enjoyed life in Austin: bicycle, swimming, concerts, theater plays, the bats under the bridge on the lake, ... Too much to give here

[16]This corresponds to the habit in probability theory, of describing the law of a process $(X(t))_{t \in T}$ by giving the *marginals*, that is the law of random vectors $(X(t_1) \dots X(t_n)) \in \mathbb{R}^n$ extracted from the random process.

[17]Y. Choquet-Bruhat and C. DeWitt-Morette, *Analysis, Manifolds and Physics*, Part I: *Basics* and part II: *92 applications*, North Holland, 1982 and 1989.

[18]For a technical description of this book, see the companion note "A tutorial in Feynman path integrals".

[19]B. DeWitt, *The global approach to Quantum Field Theory*, Oxford Univ. Press, 2004.

details. For all that, including the rewarding enterprise of writing a book:

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