Gravitational radiation from inspiralling compact binaries completed at the third post-Newtonian order

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Gravitational radiation from inspiralling compact binaries completed at the third post-Newtonian order

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The gravitational radiation from point particle binaries is computed at the third post-Newtonian (3PN) approximation of general relativity. Three previously introduced ambiguity parameters, coming from the Hadamard self-field regularization of the 3PN source-type mass quadrupole moment, are consistently determined by means of dimensional regularization, and proved to have the values $\xi = -9871/9240$, $\kappa = 0$ and $\zeta = -7/33$. These results complete the derivation of the general relativistic prediction for compact binary inspiral up to 3.5PN order, and should be of use for searching and deciphering the signals in the current network of gravitational wave detectors.

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Astrophysical systems known as inspiralling compact binaries (ICBs) — two neutron stars or black holes driven into coalescence by emission of gravitational radiation — are prominent observable sources for the gravitational wave observatories LIGO and VIRGO. The appropriate theoretical description of ICBs is by two structureless point-particles, characterized solely by their masses m_1 or m_2 (and possibly their spins), and moving on a quasi-circular orbit. Strategies to detect and analyze the very weak signals from compact binary inspiral involve matched filtering of a set of accurate theoretical template waveforms against the output of the detectors. Several analyses [1, 2, 3] have shown that, in order to get sufficiently accurate theoretical templates, one must include high-order post-Newtonian effects, up to the third post-Newtonian approximation (3PN, or $\sim 1/c^6$, where c denotes the speed of light), or even better the 3.5PN $\sim 1/c^7$ one. To date, the templates have been completed through 2.5PN order (for both the phase [4] and amplitude [5]), and the specific effects of gravitational wave tails, and tails generated by tails themselves, have been added up to the 3.5PN order [6].

Up to now the 3PN-accurate radiation field has only been incompletely determined. Previous work at 3PN order showed the appearance of "*ambiguity parameters*", due to an incompleteness of the Hadamard regularization (HR) employed for curing the infinite self field of point particles. By ambiguity parameter we mean an arbitrary dimensionless coefficient whose value cannot be fixed within HR. In the binary's 3PN Arnowitt-Deser-Misner Hamiltonian [7] there initially appeared two ambiguities, the "kinetic" ambiguity ω_k and the "static" one ω_s, while in the 3PN equations of motion in harmonic coordinates [8] there appeared a single ambiguity parameter λ turning out to be equivalent to ω_s. The kinetic ambiguity could be resolved by imposing the global Poincaré invariance of the formalism [8, 9]. The ADM Hamiltonian and harmonic-coordinates equations of motion have been shown to yield completely equivalent results [10, 11]. More recent work using *dimensional regularization* (DR) finally determined the static ambiguity to the value $\omega_s = 0$ [12] or, equivalently, $\lambda = -1987/3080$ [13]. The same result was also obtained by means of a surface-integral approach [14].

We are concerned here with the problem of the binary's 3PN radiation field (beyond the Newtonian quadrupole formalism), for which *three* ambiguity parameters, ξ, κ, ζ, have been shown to appear, coming from the HR of the source-type mass quadrupole moment I_{ij} of point particle binaries at 3PN order [15]. The terms corresponding to these ambiguities are given as follows (see Eq. (10.25) in [15]),

$$\Delta I_{ij}[\xi,\,\kappa,\,\zeta] \;\equiv\; \frac{44}{3}\frac{G_N^2 m_1^3}{c^6}\left[\left(\xi+\kappa\frac{m_1+m_2}{m_1}\right)y_1^{\langle i}a_1^{j\rangle} + \zeta v_1^{\langle i}v_1^{j\rangle}\right] + 1 \leftrightarrow 2\,. \qquad (1)$$

Here, G_N is Newton's constant, the factor $1/c^6$ indicates the 3PN approximation, and $\mathbf{y}_1$, $\mathbf{v}_1$, $\mathbf{a}_1$ denote the first particle's position, velocity and acceleration. The symbol $1 \leftrightarrow 2$ means the same terms but with all particles' labels exchanged; the brackets $\langle\rangle$ surrounding indices refer to the symmetric-trace-free (STF) projection.

In this Letter we present, for the first time, the values of the parameters ξ, κ and ζ, and we outline their derivation using DR (our detailed investigation will be reported in separate papers [16, 17, 18]). The main strategy is to express both the HR and DR results in terms of their "core" part, obtained by applying the so-called "pure-Hadamard-Schwartz" (pHS) regularization. (Following the definition of [13], the pHS regularization is a specific, minimal Hadamard-type regularization of integrals, together with a minimal treatment of "contact" ambiguities, and the use of Schwartz distributional derivatives.) The first step of our calculation [16] is to relate the final

HR 3PN quadrupole moment, for general orbits, to its pHS part:

$$I_{ij}^{(\mathrm{HR})}[r'_1, r'_2; \xi, \kappa, \zeta] = I_{ij}^{(\mathrm{pHS})}[r'_1, r'_2] + \Delta I_{ij}[\xi + \tfrac{1}{22}, \kappa, \zeta + \tfrac{9}{110}]\,. \quad (2)$$

Here, the left-hand side (L.H.S.) denotes the (non-circular generalization of the) result of [15], while the right-hand side (R.H.S.) contains both the "core" pHS quadrupole moment, and the effect of adding the ambiguities (with some numerical shifts coming from the difference between the hybrid Hadamard-type regularization scheme used in [15] and the pHS one). The pHS part is free of ambiguity parameters but depends on the specific regularization length scales r'_1 and r'_2 introduced in the harmonic-coordinates equations of motion [8].

The next step is to derive the multipole moments of an isolated (slowly-moving) source in d spatial dimensions in order to apply DR [18]. The Einstein field equations in $d+1$ space-time dimensions are "relaxed" by means of the condition of harmonic coordinates, $\partial_\nu h^{\mu\nu} = 0$, where the gravitational field variable is defined by $h^{\mu\nu} \equiv \sqrt{g}\, g^{\mu\nu} - \eta^{\mu\nu}$, with $g^{\mu\nu}$ being the inverse and $-g$ the determinant of the usual covariant metric, and with $\eta^{\mu\nu}$ the Minkowski metric (in Minkowskian coordinates). Then,

$$\Box_\eta\, h^{\mu\nu} = \frac{16\pi G}{c^4}\, g\, T^{\mu\nu} + \Lambda^{\mu\nu}[h] \equiv \frac{16\pi G}{c^4}\, \tau^{\mu\nu}\,, \qquad (3)$$

where $\Box_\eta$ denotes the flat space-time d'Alembertian operator, $T^{\mu\nu}$ the matter stress-energy tensor, $\Lambda^{\mu\nu}$ the effective gravitational source term (non-linearly depending on $h^{\rho\sigma}$ and its space-time derivatives), and $\tau^{\mu\nu}$ the total stress-energy pseudo tensor of the matter and gravitational fields. G is related to the usual three-dimensional Newton's constant G_N by $G = G_N\, \ell_0^{d-3}$, where ℓ_0 denotes an arbitrary length scale. We have obtained the mass and current multipole moments (I_L and J_L) of an arbitrary post-Newtonian source, generalizing the three-dimensional expressions derived in [19] to any d dimensions. The moments I_L and J_L parametrize the linearized approximation in the multipolar-post-Minkowskian metric *exterior* to an isolated source [20]. In the case of the mass-type moments we find

$$\begin{aligned} I_L(t) = \frac{d-1}{2(d-2)}\,\mathrm{FP}_B \int d^d\mathbf{x} \left(\frac{|\mathbf{x}|}{r_0}\right)^B \Bigg\{ &\hat{x}_L\, \Sigma_{[\ell]} - \frac{4(d+2\ell-2)}{c^2(d+\ell-2)(d+2\ell)}\,\hat{x}_{aL}\,\Sigma^{(1)}_{[\ell+1]a} \\ &+ \frac{2(d+2\ell-2)}{c^4(d+\ell-1)(d+\ell-2)(d+2\ell+2)} \\ &\times \hat{x}_{abL}\,\Sigma^{(2)}_{[\ell+2]ab} \Bigg\}(\mathbf{x}, t)\,, \end{aligned} \qquad (4)$$

where $L \equiv i_1 \cdots i_\ell$ is a multi-index composed of ℓ spatial indices ($\ell \geq 2$ is the multipolar order), $\hat{x}_L$ is the STF part of the product of ℓ spatial vectors [*i.e.*, $\hat{x}_L \equiv \mathrm{STF}(x_{i_1} \cdots x_{i_\ell})$], and the time derivatives are denoted by a superscript (n). The integrand in (4) is made out of the source densities $\Sigma\, c^2 \equiv 2[(d-2)\overline{\tau}^{00} + \overline{\tau}^{ss}]/(d-1)$, $\Sigma_a\, c \equiv \overline{\tau}^{a0}$ and $\Sigma_{ab} \equiv \overline{\tau}^{ab}$, built from the formal *post-Newtonian* expansion, denoted $\overline{\tau}^{\mu\nu}$, of the pseudo tensor $\tau^{\mu\nu}$. For any of these source densities the subscript $[\ell]$ denotes the infinite post-Newtonian-type expansion $\Sigma_{[\ell]}(\mathbf{x}, t) \equiv \sum_{k=0}^{+\infty} \alpha_\ell^k (|\mathbf{x}|/c)^{2k} \Sigma^{(2k)}(\mathbf{x}, t)$, where the coefficients are related to the Eulerian Γ-function by $\alpha_\ell^k = \Gamma(\frac{d}{2}+\ell)/[2^{2k} k!\, \Gamma(\frac{d}{2}+\ell+k)]$. The expression (4) involves a regularization factor $(|\mathbf{x}|/r_0)^B$, where $B \in \mathbb{C}$ and r_0 is a separate "infra-red" (IR) length scale, and a particular process of taking the finite part (FP_B), which constitutes the appropriate d-dimensional generalization of the finite part process used in [19] to treat the IR divergencies of the PN-expanded multipole moments (linked to the region $|\mathrm{x}| \to +\infty$).

Since the source densities Σ, Σ_a and Σ_{ab} depend on the post-Newtonian expansion of the metric, $\overline{h}^{\mu\nu}$, they are obviously to be iterated in a post-Newtonian way in order to obtain a useful result. At the 3PN order it is convenient to parametrize the moments by means of the explicit retarded potentials V, V_a, $\hat{W}_{ab}$, $\hat{R}_a$ and $\hat{X}$, introduced when $d = 3$ in [8] and generalized to d dimensions in [13]. Starting from the matter source densities $\sigma c^2 \equiv 2[(d-2)T^{00} + T^{ss}]/(d-1)$, $\sigma_a c \equiv T^{a0}$ and $\sigma_{ab} \equiv T^{ab}$, we first define the "linear" potentials $V = \Box_R^{-1}[-4\pi G\sigma]$ and $V_a = \Box_R^{-1}[-4\pi G\sigma_a]$, where $\Box_R^{-1}$ is the usual flat space-time retarded operator. The linear potentials are then used to construct higher "non-linear" potentials, such as $\hat{W}_{ab} = \Box_R^{-1}[-4\pi G(\sigma_{ab} - \frac{1}{d-2}\,\delta_{ab}\,\sigma_{ss}) - \frac{d-1}{2(d-2)}\,\partial_a V \partial_b V]$. The retardations in the latter potentials are systematically expanded to the required PN order. At Newtonian order the expression (4) reduces to the standard result $I_L = \int d^d\mathbf{x}\, \rho\, \hat{x}_L + \mathcal{O}(c^{-2})$ with $\rho = T^{00}/c^2$.

We have used Eq. (4) to compute the *difference* between the DR result and the pHS one [18]. As in the work on equations of motion [12, 13], we find that the ambiguities arise solely from the terms in the integration regions near the particles ($r_1 = |\mathbf{x}-\mathbf{y}_1| \to 0$ or $r_2 = |\mathbf{x}-\mathbf{y}_2| \to 0$) that give rise to poles $\propto 1/\varepsilon$ (where $\varepsilon \equiv d-3$), corresponding in 3 dimensions to logarithmic ultraviolet (UV) divergencies. We have verified (thanks to the appropriate definition of the finite part process FP_B) that the IR region ($|\mathrm{x}| \to +\infty$) did not contribute to the difference DR − pHS. The "compact-support" terms in the integrand of (4), *i.e.*, the terms proportional to the matter source densities σ, σ_a and σ_{ab}, were also found not to contribute to the difference (thanks to the definition of "contact" terms in pHS [13]). We are therefore left with evaluating the difference linked with the computation of the *non-compact* terms in the *local* expansion of the integrand in (4) near the singularities (*i.e.*, $r_1 \to 0$ and $r_2 \to 0$) that produce poles in d dimensions. Let us denote by $F^{(d)}$ the non-compact part of the integrand of the d-dimensional multipole moment (4) (including the appropriate multipolar factors such as $\hat{x}_L$), that is to say we write the non-compact part of I_L as the integral $\int d^d\mathbf{x}\, F^{(d)}$, extended, say, over two small domains $0 < r_1 < \mathcal{R}_1$, $0 < r_2 < \mathcal{R}_2$. [At this stage we can set $B = 0$ in $F^{(d)}$, and remove the FP_B prescription.] We write the expansion of $F^{(d)}$ when $r_1 \to 0$, up to any order N, in the form (*cf.* [13])

$$F^{(d)}(\mathbf{x}) = \sum_{p,q} r_1^{p+q\varepsilon}\,{}_1f^{(\varepsilon)}_{p,q}(\mathbf{n}_1) + o(r_1^N)\,, \qquad (5)$$

where p, q are relative integers, and $\mathbf{n}_1 \equiv (\mathbf{x} - \mathbf{y}_1)/r_1$.

In practice the ${}_1f^{(\varepsilon)}_{p,q}$'s are computed by specializing the general expressions of the non-linear retarded potentials V, V_a, $\hat{W}_{ab}$, $\cdots$ (valid for general extended sources) to the point particles case in d dimensions. The matter source density reads $\sigma = \tilde{\mu}_1(t)\,\delta^{(d)}(\mathbf{x}-\mathbf{y}_1)+1 \leftrightarrow 2$, where $\delta^{(d)}$ is Dirac's delta-function in d dimensions, and $\tilde{\mu}_1(t)$ denotes a certain function of time, coming from the standard prescription for point particles in general relativity, and which is computed in an iterative post-Newtonian way. The function $\tilde{\mu}_1$ depends on the potentials V, V_a, $\cdots$, evaluated at the location of the singular points following the rules of DR, *i.e.*, by invoking analytic continuation in $d \in \mathbb{C}$. We PN expand the (time-symmetric) propagators. For instance, at the 1PN order, we use $\Box^{-1} = \Delta^{-1} + \frac{1}{c^2}\Delta^{-2}\partial_t^2 + \mathcal{O}(c^{-4})$, which yields the solution $V = G\,\tilde{\mu}_1\,\tilde{k}\,r_1^{2-d} + \partial_t^2\big[G\,\tilde{\mu}_1\,\tilde{k}\,r_1^{4-d}/2c^2(4-d)\big] + 1 \leftrightarrow 2+\mathcal{O}(c^{-4})$, where we denote $\tilde{k} \equiv \Gamma(\frac{d-2}{2})/\pi^{\frac{d-2}{2}}$. Proceeding further, we insert the previous solution for V into the quadratic part of the source term for $\hat{W}_{ab}$ (whose structure is $\sim \partial V \partial V$), expand it when $r_1 \to 0$, and then integrate *term by term* in order to find the local expansion (when $r_1 \to 0$) of the corresponding solution, using the integration formula $\Delta^{-1}\big[\hat{n}_1^L\, r_1^\alpha\big] = \hat{n}_1^L\, r_1^{\alpha+2}/\big[(\alpha-\ell+2)(\alpha+\ell+d)\big]$. We generate by this method a *particular* solution of the Poisson-like equation we want to solve, and we added the supplementary homogeneous solution defined in Ref. [13], whose programs we have re-used for the present work.

The difference $\mathcal{D}I$ between the DR evaluation of the (d-dimensional) local integral $\int d^d\mathbf{x}\, F^{(d)}(\mathbf{x})$, and its corresponding, three-dimensional pHS evaluation, *i.e.*, the "partie finie" $\mathrm{Pf}_{s_1,s_2}\int d^3\mathbf{x}\, F^{(d=3)}(\mathbf{x})$, is expressible in terms of the expansion coefficients of (5) as (see also [12])

$$\begin{aligned}\mathcal{D}I[s_1, s_2; \varepsilon, \ell_0] \;=\; & \frac{\Omega_{2+\varepsilon}}{\varepsilon}\sum_q \left[\frac{1}{q+1} + \varepsilon \ln s_1\right] \\ & \times \langle\, {}_1f^{(\varepsilon)}_{-3,q}(\mathbf{n}_1)\,\rangle + 1 \leftrightarrow 2 + \mathcal{O}(\varepsilon)\,, \quad (6)\end{aligned}$$

where the brackets denote the angular average over the unit sphere in $3+\varepsilon$ dimensions (with total volume $\Omega_{2+\varepsilon}$) centered on the singularity $\mathbf{y}_1$. The L.H.S. depends both on the regularization length scales s_1, s_2 of the Hadamard partie finie, and on the DR regularization characteristics, $\varepsilon = d - 3$ and ℓ_0.

With this definition, the dimensional regularization of the 3PN quadrupole moment (indices $L \equiv ij$) is obtained as the sum of its "pure-Hadamard-Schwartz" part, and of a "difference" computed according to Eq. (6): $I^{(\mathrm{DR})}_{ij}[\varepsilon, \ell_0] = I^{(\mathrm{pHS})}_{ij}[s_1, s_2] + \mathcal{D}I_{ij}[s_1, s_2; \varepsilon, \ell_0]$. At this stage a check of the result is that the HR scales s_1 and s_2 cancel out between the two terms in the R.H.S., so that $I^{(\mathrm{DR})}_{ij}$ depends only on ε and ℓ_0 [its dependence on ε is of the form of a simple pole followed by a finite part, $a_{-1}\,\varepsilon^{-1} + a_0 + \mathcal{O}(\varepsilon)$]. Because of this independence from s_1, s_2, we can express our result in terms of the constants r'_1 and r'_2 used as fiducial scales in the final result of [15, 16], see Eq. (2). We can therefore write the DR result as

$$I^{(\mathrm{DR})}_{ij}[\varepsilon, \ell_0] = I^{(\mathrm{pHS})}_{ij}[r'_1, r'_2] + \mathcal{D}I_{ij}[r'_1, r'_2; \varepsilon, \ell_0]\,. \qquad (7)$$

Let us now impose the *physical equivalence* between the DR result (7) and the corresponding HR result (2). In doing this identification, we must remember that the "bare particle positions", $\mathbf{y}_1^{\mathrm{bare}}$ and $\mathbf{y}_2^{\mathrm{bare}}$, entering the DR result differ from their Hadamard counterparts (used in [8, 15, 16]) by some *shifts*, which were uniquely determined in [13], and denoted there $\boldsymbol{\xi}_1(r'_1; \varepsilon, \ell_0)$ and $\boldsymbol{\xi}_2(r'_2; \varepsilon, \ell_0)$ (see Eqs. (1.13) and (6.41)–(6.43) in [13]). In the present work, we will denote them by $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ in order to avoid any confusion with the ambiguity parameter ξ. In other words, we impose the equivalence

$$I^{(\mathrm{HR})}_{ij}[r'_1, r'_2; \xi, \kappa, \zeta] = \lim_{\varepsilon \to 0}\left[I^{(\mathrm{DR})}_{ij}[\varepsilon, \ell_0] + \delta_{\boldsymbol{\eta}(r'_1, r'_2; \varepsilon, \ell_0)} I_{ij}\right]\,, \qquad (8)$$

in which $\delta_{\boldsymbol{\eta}} I_{ij} = 2\, m_1\, y_1^{<i}\eta_1^{j>} + 1 \leftrightarrow 2$ denotes the total change induced on the quadrupole moment by the latter shifts. We find that the poles $\sim 1/\varepsilon$ separately present in the two terms in the brackets of (8) cancel, so that the physical ("dressed") DR quadrupole moment is *finite* and given by the limit shown in (8).

Finally, by inserting the expressions of the DR and HR results given respectively by (7) and (2) into Eq. (8), and by removing the pHS part which is common to both results, we obtain a relation for the ambiguity part ΔI_{ij} of the quadrupole moment in (2) in terms of known quantities,

$$\begin{aligned}\Delta I_{ij}[\xi + \tfrac{1}{22}, \kappa, \zeta + \tfrac{9}{110}] \;=\; & \lim_{\varepsilon \to 0}\big[\mathcal{D}I_{ij}[r'_1, r'_2; \varepsilon, \ell_0] \\ & + \delta_{\boldsymbol{\eta}(r'_1, r'_2; \varepsilon, \ell_0)} I_{ij}\big]\,. \quad (9)\end{aligned}$$

The apparent dependence of the R.H.S. on r'_1, r'_2, ε and ℓ_0, is checked to cancel out. Equation (9) then gives *three* equations for the three unknowns ξ, κ, ζ, thereby yielding the central result of this work:

$$\xi = -\frac{9871}{9240}\,, \quad \kappa = 0\,, \quad \zeta = -\frac{7}{33}\,, \qquad (10)$$

which finally provides an unambiguous determination of the 3PN radiation field by DR.

We have been able to perform several checks of our calculation. First of all, we have also obtained ζ by considering the limiting physical situation where the mass of one of the particles is exactly zero (say $m_2 = 0$), and the other particle moves with uniform velocity. Computing the quadrupole moment of a *boosted* Schwarzschild black hole at 3PN order and comparing the result with $I_{ij}^{(\mathrm{HR})}$ in the limit $m_2 = 0$ we recover exactly the same value for ζ [17]. This agreement is a direct verification of the global Poincaré invariance of the wave generation formalism, and a check that DR automatically preserves this invariance (as it did in the context of the equations of motion [12, 13]). Secondly, the fact that the value of κ is zero has been checked by showing that there are no corresponding dangerously divergent "diagrams" (in the sense of [21]). Finally, we have computed the DR of the mass *dipole* moment, $I_i^{(\mathrm{DR})}$, following the same method and using the same computer programs, and found that it differs by exactly the same shifting of the world-lines, $\boldsymbol{\eta}_1(r_1'; \varepsilon, \ell_0)$ and $\boldsymbol{\eta}_2(r_2'; \varepsilon, \ell_0)$, as in Eq. (8), *i.e.*, $\delta_{\boldsymbol{\eta}} I_i = m_1\, \eta_1^i + 1 \leftrightarrow 2$, from the 3PN center-of-mass position associated with the equations of motion in harmonic coordinates as given by Eq. (4.5) of [11].

The determination of the values (10) completes the problem of the general relativistic prediction for the templates of ICBs up to 3PN order (and actually up to 3.5PN as the corresponding "tail terms" have already been determined [6]). The relevant combination of the parameters (10) entering the 3PN energy flux in the case of circular orbits, namely θ [15], is now fixed to

$$\theta \equiv \xi + 2\kappa + \zeta = -\frac{11831}{9240}\,. \tag{11}$$

Numerically, $\theta \simeq -1.28041$. The orbital phase of compact binaries, in the adiabatic inspiral regime (*i.e.*, evolving by radiation reaction), involves at 3PN order a linear combination of θ and of the equation-of-motion related parameter λ [22], which is determined as

$$\hat{\theta} \equiv \theta - \frac{7}{3}\lambda = \frac{1039}{4620}\,. \tag{12}$$

The fact that the numerical value of this parameter is quite small, $\hat{\theta} \simeq 0.22489$, indicates, following measurement-accuracy analyses [3], that the 3PN (or better 3.5PN) order should provide an excellent approximation for both the on-line search and the subsequent off-line analysis of gravitational wave signals from ICBs in the LIGO and VIRGO detectors.

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