IR Free or Interacting? A Proposed Diagnostic

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arXiv:hep-th/0509085 v1   12 Sep 2005

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We present and discuss a conjectured criterion for determining whether a 4d quantum field theory is IR free, or flows to an interacting conformal field theory in the infrared: "the correct infrared phase is that with the larger conformal anomaly a". A stronger conjecture is that "an operator can become IR free only if that results in a larger conformal anomaly a". We test these conjectures in the context of $\mathcal{N} = 1$ supersymmetric theories. They are verified to indeed predict the correct IR phase in every tested case, for a plethora of examples for which the infrared phase could already be determined on other grounds. When applied to the still unsettled case of $SU(2)$ with a chiral superfield in the isospin $3/2$ representation, the conjecture suggest that the IR phase is conformal rather than confining.

September 2005

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[1] Sabbatical address, Sept. '05 to June '06.

1. Introduction

4d asymptotically free gauge theories have various possible IR phases. In the extreme IR, the massive degrees of freedom decouple, and remaining, massless degrees of freedom are either IR free, or they're an interacting conformal field theory. As examples, pure Yang Mills confines, with no massless degrees of freedom left in the IR. $SU(N_c)$ QCD with N_f massless quark flavors, provided that N_f is sufficiently small, has confinement and $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_D$ chiral symmetry breaking; the theory in the extreme IR contains just the $N_f^2 - 1$ massless, Goldstone boson pions with interactions, given by the chiral Lagrangian, that are all irrelevant in the extreme IR. For N_f sufficiently large, on the other hand, so that the theory is just barely asymptotically free, the IR theory is an interacting conformal field theory – the Banks-Zaks RG fixed point – corresponding to the zero of the perturbative beta function at small coupling in this case [1,2].

These examples illustrate a general intuition. When the massless matter content is closer to (but still below) the asymptotic freedom bound, the IR phase is likely interacting conformal. When the massless matter content is farther below the asymptotic freedom bound, the coupling runs to larger values, and the IR phase is then more likely to be an IR free theory of confined composites. A long standing goal is to make this intuition more precise. Since we're unable to analytically solve the theory in the IR, the options are to either to go to the lattice, or to develop and employ various diagnostics and strong coupling methods, to try to gain insight into the IR phase. This latter route (a.k.a. "voodoo QCD") can be used to make predictions, but one can not be absolutely certain as to whether or not they are correct. An example of a strong coupling method are the gap equations, which when applied to $SU(N_c)$ QCD with N_f flavors suggest that the theory has confinement and chiral symmetry breaking for all N_f below a critical value N_f^c, and is conformal for all N_f in the range $N_f^c < N_f < 11N_c/2$, with the critical value estimated to be $N_f^c \approx 4N_c$ [3]. See e.g. [4,5] for recent further analysis of this case.

Confinement can occur with, or without chiral symmetry breaking, depending on the gauge group and matter content (as exhibited by Seiberg in the context of $\mathcal{N} = 1$ supersymmetric SQCD [6]). 't Hooft anomaly matching [7] strongly constrains the IR phase of theories with unbroken chiral symmetries: because the 't Hooft anomalies of global symmetries are unchanged along the RG flow, the IR free spectrum of confined composites must contain massless fermions, that must match the 't Hooft anomalies of the UV theory. (When the chiral symmetry is broken, the Goldstone bosons instead saturate

the 't Hooft anomalies, via a Wess-Zumino term.) A non-trivial, confining, solution of 't Hooft anomaly matching can be regarded as some kind of non-trivial evidence for the confining scenario. There are many examples of this in the context of supersymmetric gauge theories. One particular, and still unsettled, example that we'll discuss further in what follows is $\mathcal{N} = 1$ supersymmetric $SU(2)$ gauge theory with a matter chiral superfield in the $I = 3/2$ representation [8]. But it was pointed out in [9] that there are other examples, where highly non-trivial 't Hooft anomaly matching, suggesting IR free confinement, can be a misleading fluke – and the IR theory is instead interacting conformal.

Another diagnostic for the IR phase is the conjecture that the number of massless degrees of freedom, suitably defined, decreases in RG flows to the IR, in analogy with Zamolodchikov's c-theorem for 2d quantum field theories [10]. We'll here be specifically interested in Cardy's conjectural a-theorem [11], that the conformal anomaly coefficient a (the coefficient of the Euler density term in $\langle T^\mu_\mu \rangle$ when the theory is put on a fixed, curved, spacetime background) satisfies $a_{UV} > a_{IR}$ for all 4d RG flows (and also $a_{IR} \geq 0$). There is not yet a definitive proof of Cardy's conjecture, but there is a huge amount of non-trivial evidence for it, and not yet a single counter-example. The evidence is especially compelling in the context of supersymmetric gauge theories, where the conformal anomaly a can be computed exactly, even for interacting conformal field theories – see e.g. [12-20].

The successes of Cardy's conjecture are especially impressive when one or both of the endpoints of the RG flow are interacting. It's successful, but in a less impressive way, when applied to RG flows with asymptotically free quarks and gluons in the UV, and free confined composites in the IR. E.g. for $SU(N_c)$ with N_f quark flavors, the RG flow from asymptotically free quarks and gluons in the UV, to IR free pions in the IR, has

$$
\begin{aligned}
a_{UV} &= (N_c^2 - 1)a_1^{free} + 2N_c N_f a_{1/2}^{free} \\
a_{IR}^{confine} &= (N_f^2 - 1)a_0^{free}.
\end{aligned}
\tag{1.1}
$$

Here a_j^{free} is the conformal anomaly coefficient a for a free field of spin j; which are found (see e.g. in [21] and references cited therein) to be given by $a_1^{free} = 62a_0^{free}$ for a massless spin 1 field and $a_{1/2}^{free} = \frac{11}{2}a_0^{free}$ for a massless 2-component fermion. For the case (1.1), $a_{UV} > a_{IR}$ is easily satisfied for any asymptotically free N_f, so here Cardy's conjecture does not put any interesting constraint on the maximum N_f for which confinement and chiral symmetry breaking occurs. Applied in this way, Cardy's conjecture does not help much in determining the IR phase of the theory.

In this paper, we will propose and test a more conjectural diagnostic for determining the IR phase of a given theory. The idea is to compute the conformal anomaly coefficient a in both the IR free phase scenario, and also in the interacting conformal field theory phase scenario. Our conjectural diagnostic is that *the correct IR phase is the one with the **larger** value of the conformal anomaly a.* The motivation is that Cardy's conjecture suggests that $\Delta a \equiv a_{UV} - a_{IR}$ is some measure of the RG distance between theories (see e.g. the discussion in [22]). So the conjecture that we'll explore here can be thought of as saying that the preferred IR phase is the one that's closest, in terms of RG distance, to the UV starting point. A perhaps reasonable objection to the conjecture is that the IR theory is either interacting conformal or IR free – either it hits a zero of the beta functions before confinement sets in, or it does not – so it is not clear whether or not there is any meaning in comparing them, as if they were both viable possibilities. In any case, we will show phenomenologically, by considering many understood examples, that the proposal correctly predicts the IR phase in every case.

Applied to $SU(N_c)$ QCD with N_f massless quark flavors, the idea would be to compute

$$
\begin{aligned}
a_{IR}^{confine} &\equiv a_{IR}^{free} = (N_f^2 - 1)a_0^{free} \\
a_{IR}^{interacting} &= (N_c^2 - 1)a_1^{free} + 2N_cN_f a_{1/2}^{free} - (\text{interaction contributions}).
\end{aligned}
\tag{1.2}
$$

The interaction contributions in the conformal phase come from the presumed non-zero RG fixed point value of the gauge coupling, g_*. As indicated in (1.2), the interactions are expected, from Cardy's conjecture, to always decrease the conformal anomaly a – this can be verified perturbatively for Banks-Zaks RG fixed points. In the Banks-Zaks limit of large, barely asympotically free N_f, the interaction contributions in (1.2) are $O(g_*^2) \ll 1$, and then comparing the two alternatives in (1.2) we find $a_{IR}^{interact} > a_{IR}^{free}$, so our diagnostic correctly predicts the interacting conformal phase in this limit. (This is a trivial success: the Banks-Zaks fixed point has a small value of $\Delta a \equiv a_{UV} - a_{IR}$ by assumption, and a_{IR}^{free} couldn't have exceeded $a_{IR}^{interacting}$ in this limit without violating Cardy's conjecture.) Now consider reducing N_f: this increases the RG fixed point coupling g_*, and by Cardy's conjecture we expect that the interaction contributions to $a_{IR}^{interacting}$ become more and more important, and always tend to further reduce the value of $a_{IR}^{interacting}$. It's possible, then, that there is a critical value N_f^c where the interaction effects are sufficiently strong so that $a_{IR}^{interacting}$ drops below the value a_{IR}^{free} in (1.2); our proposed diagnostic would then predict that the theory is IR free for this value of N_f and below. Unfortunately, we are unable at present to explicitly and reliably compute the interaction contributions to

$a_{IR}^{interacting}$ in the strong coupling regime for this and other non-supersymmetric theories, to compare the value of N_f^c obtained in this way with those via diagnostics [3-5,23]

In what follows, we will test and apply the proposed diagnostic in the context of 4d $\mathcal{N} = 1$ supersymmetric gauge theories, where it's possible to exactly compute both a_{IR}^{free} and $a_{IR}^{interacting}$, even at strongly interacting RG fixed points. This will be reviewed in the following section. We will also discuss a stronger version of our conjecture: "operators can only become IR free if that increases a". In section 3, we will test and apply the proposal in many examples. As we will note there, for the unsettled example [8] of $SU(2)$ with matter in the $I = 3/2$, the proposal predicts that the IR theory is an interacting RG fixed point, rather than the confining phase. But since the proposal here is only a conjecture, this of course does not yet completely settle the issue of the IR phase of that example.

2. Exact results for a in supersymmetric gauge theories

4d $\mathcal{N} = 1$ superconformal field theories have a conserved $U(1)_R$ symmetry, that's related by supersymmetry to the dilatation current. We'll denote the charges of the fields under this superconformal R-symmetry by R_*. It was shown in [12] that the conformal anomaly a_{SCFT} is related by supersymmetry to the 't Hooft anomalies of this R-symmetry:

$$a_{SCFT} = \frac{3}{32}(3\mathrm{Tr}R_*^3 - \mathrm{Tr}R_*). \tag{2.1}$$

This formula is very powerful: using the power of 't Hooft anomaly matching, it allows a_{SCFT} to be computed, even for strongly interacting IR fixed points, from the R_* charges of the weakly coupled, asymptotically free, UV spectrum.

Moreover, in [14] it was shown that the superconformal R-symmetry R_* can be exactly determined by "a-maximization": it's the local maximum of the function

$$a_{trial}(R) = \frac{3}{32}(3TrR^3 - \mathrm{Tr}R) \tag{2.2}$$

over all possible, conserved R-symmetries. The proposed diagnostic of the present paper fits with the intuition of a-maximization: the conformal anomaly a wants to be as big as possible. This helps ensure that a then decreases in RG flows to the IR [14].

As will be important in what follows, theories can develop accidental symmetries in the IR. To be concrete, suppose that a set of composite, gauge invariant operators – let's call them X – decouple in the extreme IR from the other operators, i.e. correlation functions

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involving X can be computed by treating X as decoupled, free fields. The effect of this on a_{SCFT}, and on the a-maximization procedure, can be included via 't Hooft anomaly matching [13,15]. The idea is that the contribution of the fields X to the 't Hooft anomalies in (2.1) can be isolated, using anomaly matching, as:

$$a_X(R_X) = \frac{3}{32}\dim(X)\left[3(R_X - 1)^3 - (R_X - 1)\right], \tag{2.3}$$

where $\dim(X)$ is the number of operators X and R_X is their superconformal R-charge. Without accidental symmetries, R_X would be fixed to $R_X = R^{(0)}(X)$, the naive R-charge of the composite operator, obtained by adding the R-charges of the fields in it. But when X is a free field, there is an additional $U(1)_X$ accidental symmetry, and a-maximization requires maximization over the full space of possible R-symmetries, including mixing with $U(1)_X$. This means that R_X becomes an additional variable to locally maximize over, without being constrained to equal $R^{(0)}(X)$. Maximizing (2.3) over R_X gives $R_X = 2/3$, the correct value for a free field. The upshot of this change is to replace (2.1) or (2.2) with:

$$\begin{aligned} a &\to a + a_X(2/3) - a_X(R^{(0)}(X)) \\ &= a + \frac{1}{96}\dim(X)(2 - 3R^{(0)}(X))^2(5 - 3R^{(0)}(X)). \end{aligned} \tag{2.4}$$

There is a unitarity bound $R(X) \geq 2/3$ for gauge invariant, spinless chiral primary operators X (since $\Delta X \geq 1$), with equality if and only if X is a free field. If some operator X would otherwise appear to violate this bound, the picture is that, in RG flow to the IR fixed point, $R(X)$ flows down until it hits $2/3$, and thereafter X remains free, with $R(X) = 2/3$. In this case, because $R^{(0)}(X) < 2/3$, the modification of (2.4) increases the value of a. This is reasonable: the correction in (2.4) comes from maximizing the function $a_{trial}(R)$ over a bigger space of possible trial R-symmetries, and it's reasonable that maximizing a_{trial} over the larger space of possibilities leads to a bigger maximal value.

But operators with $R^{(0)}(X) > 2/3$ can also turn out to be free fields. There can be an accidental $U(1)_X$ symmetry, which leads to the same modification (2.4), even when it is not required by the unitarity bound. In particular, in a theory that's actually confining in the IR, the confining composite operators are all IR free fields, regardless of their value of $R^{(0)}$. We will illustrate this with many examples in the following section. In this case, we see immediately from (2.4) that the $U(1)_X$ accidental symmetry leads to an increase in a_{CFT} if, and only if, $R^{(0)}(X) < 5/3$. If $R^{(0)}(X) > 5/3$, we have a peculiar situation: locally maximizing a_{trial} over the bigger space of possible R-symmetries, with

$U(1)_X$ included, then leads to a smaller value at the maximum. This is simply because, when $R^{(0)}(X) > 5/3$, locally maximizing the cubic function (2.3) over R_X will lead to a smaller value of a than if R_X were just fixed to $R^{(0)}(X)$.

Though the math is clear, having $R^{(0}(X) > 5/3$ operators become free fields runs counter to our intuition that the conformal anomaly a_{CFT} always wants to be as big as possible. This motivates the **conjecture**: *operators only become IR free if that leads to a larger value of a, i.e. operators with $R^{(0)}(X) > 5/3$ do not become IR free.* All of the understood examples, to be discussed in the following section, are compatible with this conjecture. This conjecture is a stronger version of the one in the introduction, since it states that *every* IR free operator should contribute positively to a, while the weaker conjecture is that their sum must be positive. The conjecture here can also apply to IR free operators in a theory that remains partially interacting.

3. Examples

3.1. $\mathcal{N} = 1$ $SU(N_c)$ SQCD, with N_f massless fundamental flavors

We will here be interested in the vacuum at the origin of the moduli space of vacua for $N_f > N_c$ [6,24], where the global symmetries are all unbroken. In particular, there is a conserved $U(1)_R$ symmetry, with anomaly free charges

$$R_{cons}(Q) = R_{cons}(\widetilde{Q}) = \frac{N_f - N_c}{N_f}. \tag{3.1}$$

The gauge invariant composite operators include the "mesons" $M = Q\widetilde{Q}$ and the "baryons" $B = Q^{N_c}$ and $\widetilde{B} = \widetilde{Q}^{N_c}$ (suppressing flavor and color indices), with $U(1)_R$ charges

$$R^{(0)}(M) = 2\left(\frac{N_f - N_c}{N_f}\right), \qquad R^{(0)}(B) = R^{(0)}(\widetilde{B}) = N_c\left(\frac{N_f - N_c}{N_f}\right). \tag{3.2}$$

The superscript is a reminder that these might be corrected by accidental symmetries.

Consider first $N_f = N_c + 1$ massless flavors. The vacuum at the origin has confinement without chiral symmetry breaking, with the massless spectrum consisting of [6] N_f^2 mesons M, N_f baryons B, and N_f anti-baryons $\widetilde{B}$. In the extreme IR, these confined operators are all free fields. Let us clarify this: the IR theory of [6] contained also a superpotential term for these fields, $W_{conf} \sim MB\widetilde{B} - \det M$, but any superpotential term for gauge neutral fields (other than quadratic mass terms) is always irrelevant in the extreme IR. So

the superpotential W_{conf} is irrelevant[2] in the extreme IR, $W_{conf} \to 0$, and the IR theory consists of just the confined, free massless fields $M_{i\tilde{j}}$, B^i, $\widetilde{B}^{\tilde{i}}$. The IR limit of the RG flow thus has $R_*(M) = R_*(B) = R_*(\widetilde{B}) = 2/3$, and thus, using (2.1) with this IR spectrum,

$$a_{IR}^{free} = \frac{3}{32}\left(3(R_* - 1)^3 - (R_* - 1)\right)(N_f^2 + N_f + N_f) = \frac{1}{48}(N_f^2 + 2N_f). \qquad (3.3)$$

Evaluating the 't Hooft anomalies in (2.1) using the UV asymptotically free spectrum of quarks and gluons, for general N_c, and N_f, with the conserved R-charges (3.1) yields:

$$\begin{aligned}
a^{(0)} &= \frac{3}{32}\left[2(N_c^2 - 1) + 2N_cN_f\left(3(-\frac{N_c}{N_f})^3 - (-\frac{N_c}{N_f})\right)\right], \\
&= \frac{3}{32}\left[4N_c^2 - 2 - 6\frac{N_c^4}{N_f^2}\right],
\end{aligned} \qquad (3.4)$$

where the superscript indicates that we have not yet accounted for the accidental symmetries. The same result would be obtained in a dual description, since the dual 't Hooft anomalies should match. In particular, for $N_f = N_c + 1$, the 't Hooft anomalies match between the UV and confined IR free spectrum [6], so (3.4) is also obtained upon evaluating the 't Hooft anomalies in (2.1) using the confined IR free spectrum of mesons and baryons, with R-charges (3.2). The result (3.3) is then obtained upon accounting for the accidental symmetries, using the formula (2.4) for the IR free operators M, B, $\widetilde{B}$:

$$a_{IR}^{free} = a^{(0)} + \frac{N_f^2}{96}(2-3R^{(0)}(M))^2(5-3R^{(0)}(M)) + \frac{2N_f}{96}(2-3R^{(0)}(B))^2(5-3R^{(0)}(B)). \quad (3.5)$$

Because $R^{(0)}(M) = 2/N_f < 2/3$, the unitarity bound shows that the operators M *had* to become free fields. On the other hand, the operators B and $\widetilde{B}$ have $R^{(0)}(B) = R^{(0)}(\widetilde{B}) = (N_f - 1)/N_f$, which is safely above the unitarity bound for general N_f. So there is an alternative (incorrect) scenario to the above, where rather than having all operators confine to free fields, the theory flows to an interacting conformal field theory in the IR, where the operators M are free, but the operators B and $\widetilde{B}$ are not free. In this hypothetical scenario, the value of a_{IR} for the IR theory is given by (3.4), corrected only by the accidental symmetry (2.4) for the meson fields M:

$$a_{IR}^{interacting} = a^{(0)} + \frac{N_f^2}{96}(2 - 3R^{(0)}(M))^2(5 - 3R^{(0)}(M)). \qquad (3.6)$$

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[2] More precisely, W_{conf} is "dangerously irrelevant," in that it becomes relevant upon deforming the theory, by giving the flavors masses or moving away from the origin of the moduli space of vacua. But we're only considering the undeformed theory above, so this distinction is immaterial.

Because $R^{(0)}(B) = (N_f - 1)/N_f < 5/3$, we can immediately see that the additional baryon contribution in a_{IR}^{free} (3.5), as compared with $a_{IR}^{interacting}$ (3.6), is positive, so

$$a_{IR}^{free} > a_{IR}^{interacting}. \tag{3.7}$$

Our proposed diagnostic then predicts that the actual IR phase is the IR free confining scenario, rather than the conformal one; this prediction is indeed the correct answer [6,24].

Now let us consider general numbers of flavors, in the range $N_c + 1 \leq N_f < 3N_c$, with the upper limit for asymptotic freedom. We again consider the possibility that the IR theory is an interacting conformal field theory, with the only free fields given by those gauge invariant operators which hit or violate the unitarity bound, $R(X) \geq 2/3$. Again using (3.1) for the superconformal R-charges, this gives the result

$$a_{IR}^{interacting} = a^{(0)} = \frac{3}{32}\left[4N_c^2 - 2 - 6\frac{N_c^4}{N_f^2}\right] \qquad \text{for} \quad \frac{3}{2}N_c < N_f < 3N_c,$$

$$a_{IR}^{interacting} = a^{(0)} + \frac{1}{96}N_f^2(2 - 3R^{(0)}(M))^2(5 - 3R^{(0)}(M)) \qquad \text{for} \quad N_c + 1 \leq N_f \leq \frac{3}{2}N_c, \tag{3.8}$$

where $R^{(0)}(M) = 2(N_f - N_c)/N_f$. For $N_f \leq 3N_c/2$, $R^{(0)}(M) \leq 2/3$, below the unitarity bound, so then M must be free, with the resulting additional contribution in (3.8).

For $N_f > \frac{3}{2}N_c$, the interacting conformal scenario is the only known viable possibility. But for $N_f \leq \frac{3}{2}N_c$ there is another IR phase scenario, which was argued in [24] to be the correct answer: the $SU(N_f - N_c)$ gauge fields and quarks of the non-asymptotically free, and hence IR free, magnetic dual. See e.g. [25] for some discussion of the characteristics of the free magnetic phase, e.g. the potential for sources. Here we will just use the fact that it's free in the extreme IR. The conformal anomaly of the free magnetic phase is

$$a_{IR}^{free} = a^{(0)} + \frac{1}{96}N_f^2(2 - 3R^{(0)}(M))^2(5 - 3R^{(0)}(M))$$
$$+ \frac{1}{96}2N_f(N_c - N_f)(2 - 3R^{(0)}(q))^2(5 - 3R^{(0)}(q)) \qquad \text{for} \quad N_c + 1 < N_f < \frac{3}{2}N_c. \tag{3.9}$$

Again, $a^{(0)}$ is given by (3.8), which is also obtained if the 't Hooft anomalies in (2.1) are evaluated in the magnetic dual description [24], with its conserved R-charges

$$R^{(0)}(M) = 2\left(\frac{N_f - N_c}{N_f}\right), \qquad R^{(0)}(q) = R^{(0)}(\widetilde{q}) = \frac{N_c}{N_f}, \tag{3.10}$$

by the 't Hooft anomaly matching for the dual theories [24]. The additional terms in (3.9) are to account, as in (2.4), for the fact that the field M, q, and $\widetilde{q}$ are all IR free fields in the

free magnetic scenario. Comparing (3.9) with (3.8), the only difference is in the last line of (3.9), coming from q and $\widetilde{q}$ being IR free in the free magnetic dual scenario. Because $R^{(0)}(q) = N_c/N_f < 5/3$, this additional contribution is positive, so $a_{IR}^{free} > a_{IR}^{interacting}$. Our proposed diagnostic then predicts that free magnetic scenario is preferred over the (partially) interacting conformal scenario for $N_f < \frac{3}{2}N_c$. Based on the many other duality checks in [24], this is believed to be the correct answer.

It is very similarly verified for the dualities for SO and Sp with fundamentals [26,27] that, in the free magnetic phase, the free magnetic quarks indeed satisfy $R^{(0)}(q) < 5/3$.

3.2. Other examples with IR free "confining" phases

Our conjectures can be similarly checked in the multitude of other supersymmetric examples which have been argued to have an IR free phase. The process is to compute the superconformal R-charges $R^{(0)}(X)$ of the basic gauge invariant chiral operators in a hypothetical, interacting conformal phase. Those operators with $R^{(0)}(X)$ at or below the unitarity bound are forced to be IR free in any case. Any basic operators with $R^{(0)}(X) > 2/3$ are more interesting, since they could have been interacting. Our conjecture is that, if they're actually IR free, these basic operators would have had $R^{(0)}(X) < 5/3$ in the hypothetical interacting phase. Then having them be IR free, rather than interacting, leads to larger a, and so $a_{IR}^{free} > a_{IR}^{interacting}$.

A set of examples, with t Hooft anomaly matchings suggesting an IR free spectrum, was presented in [28]. The examples with $\mu > \mu_{adj}$ (μ is the index of the matter representation and μ_{adj} that of the adjoint) will be discussed in the following subsection. The $\mu < \mu_{adj}$ examples were referred to as T1-T6 in [28], where e.g. T6 is $SO(14)$ with a single matter field in the spinor representation. Because $\mu < \mu_{adj}$, the matter fields in these examples all have $R = -(\mu_{adj} - \mu)/\mu < 0$, so all gauge invariant composite operators also have $R^{(0)}(X) < 0$. The unitarity bound then requires them to all be IR free fields. So the theories T1-T6 necessarily confine to IR free fields (on their $W_{dyn} = 0$ branch) – there is no viable interacting conformal scenario.

The "s-confining" theories of e.g. [29-34], provide many more examples. Given their many checks, we believe that these examples are indeed IR free, rather than interacting. Our conjecture can then be tested, by verifying that all of the IR free operators of every example indeed would have had $R^{(0)}(X) < 5/3$ in any viable interacting phase. Scanning the examples, this can be immediately verified in those cases where the superconformal R-symmetry is uniquely determined on symmetry grounds. For those examples with matter

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in different representations of the gauge group, on the other hand, a-maximization is needed to determine the superconformal R-charges $R^{(0)}(X)$ in the hypothetical conformal phase. This is straightforward to implement, but it would take too long here to check every possible example. Let us illustrate one example.

Consider $SU(N)$ gauge theory with matter A in the two-index anti-symmetric tensor representation $\boxed{}$, $\overline{A}$ in the conjugate representation, and N_f flavors Q_f and $\overline{Q}_f$ of fundamental and anti-fundamental matter. The confining case of [33] is $N_f = 3$, but we'll temporarily keep N_f as a free parameter. The superconformal R-charges of the fields are constrained by the anomaly free condition to satisfy $R_A = (N_f - 2 - N_f y)/(N - 2)$, with $y \equiv R_Q$. The trial a-function, before accounting for accidental symmetries, is

$$a_t^{(0)}(y) = \frac{3}{32}\left[2(N-1) + N(N-1)(3R_A^3 - 9R_A^2 + 8R_A) + 2NN_f(3(y-1)^3 - (y-1))\right]. \tag{3.11}$$

As an interesting limit, let us consider the situation for $N \gg 1$, and simply the formulae by keeping only leading order in large N. So $R_A \approx (N_f - 2 - N_f y)/N$ and (3.11) becomes

$$a_t^{(0)} \approx \frac{3N}{16}\left[5N_f(1-y) - 7 + 3N_f(y-1)^3\right]. \tag{3.12}$$

If we were to maximize this, setting $N_f = 3$, we'd obtain $y^{(0)} = 1 - \sqrt{15/27} \approx 0.255$. But we must correct, as in (2.4), for the many gauge invariant operators which are at or below the unitarity bound, and hence actually free. The gauge invariant operators are listed in case 3.1.4 or 3.1.5 of [33] for odd or even N, respectively; because N is large, there is no significant difference here between the even and odd N cases. To be comparable with (3.12), which is order N, the only significant accidental symmetry contributions are those coming from summing a series of operator contributions, i.e. the operators $M_j = Q(A\overline{A})^{j-1}\overline{Q}$, $H_j = \overline{A}(A\overline{A})^{j-1}Q^2$, $\overline{H}_j = A(A\overline{A})^{j-1}\overline{Q}^2$, and $T_j = (A\overline{A})^j$, for $j = 1 \ldots N/2$. The sum of terms (2.4) can then be approximated by an integral, as in [15,19]

$$\frac{1}{96}\sum_{j=1}^{p}[2 - 3R_j]^2[5 - 3R_j] \approx \frac{1}{288\beta}\int_{2-3R_p}^{2-3\alpha} u^2(3+u)du, \tag{3.13}$$

where α and β are defined by $R_j \equiv \alpha + (j-1)\beta$ (and $u \equiv 2 - 3R_j$), and p is either solved for by setting $R_p = 2/3$, or if all the operators are below the unitarity bound, so $R_p = 2/3$ has no solution, then $p = j_{max} = N/2$. Applying this for the operators M_j, H_j, $\overline{H}_j$ and T_j, with $N_f = 3$, modifies (3.12) to

$$a_t(y) \approx a_t^{(0)}(y) + \frac{15}{48}N(1-3y)^2(1-y) + \frac{1}{768}N(17 + 27y + 243y^2 + 729y^3). \tag{3.14}$$

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Maximizing this w.r.t. y gives $y_* = 1/3$, which implies that *every* operator is at, or below the unitarity bound. So this particular class of examples actually has no viable alternative, interacting conformal field theory scenario: the IR theory is necessarily free.

Yet another class of examples of theories is $SU(N)$ gauge theory, with an adjoint X, F fundamentals and anti-fundamentals Q and $\overline{Q}$, and superpotential

$$W_{tree} = h\mathrm{Tr}X^{k+1}, \qquad (3.15)$$

which has been argued to confine [35,36] when $N = kF - 1$, using the duality of [37,38]. We know from the a-maximization analysis [15] of the interacting SCFT with $W_{tree} = 0$ that the superpotential (3.15) is relevant, for all k, given the above relation between the colors and flavors. Thanks to the superpotential (3.15), we do not need to use a-maximization to determine the superconformal R-charges of a hypothetical conformal phase, we simply have $R(X) = 2/(k+1)$ and $R(Q) = R(\overline{Q}) = 1 - 2(kF-1)/(k+1)F$. Using these, we compute the R-charges $R^{(0)}(X)$ of the confined spectrum of operators. The generalized mesons $M_i = (\overline{Q}X^{i-1}Q)$, with $i = 1\ldots k$, are below the unitarity bound, and hence IR free in any case. The generalized baryons, $B = (Q^F(XQ)^F\ldots(X^{k-1}Q)^{F-1})$, and $\overline{B} = (\overline{Q}^F(X\overline{Q})^F\ldots(X^{k-1}\overline{Q})^{F-1})$, on the other hand, have $R^{(0)}(B) = 1 - (2/F(k+1))$, which is generally above the unitarity bound, and thus there is an alternative scenario where they're interacting. Because $R^{(0)}(B) < 5/3$, our diagnostic correctly predicts they're actually IR free. Note that the operators $T_j = \mathrm{Tr}X^j$ are not included, because they are not IR free massless fields – they pair up with quadratic superpotential mass terms.

There are many generalizations of this example, discussed in [36,39], which can be argued to confine using the dualities of [40,41,42]. Our conjecture that all IR free massless operators would have had $R^{(0)}(X) < 5/3$ in a hypothetical interacting conformal scenario can be checked in all of these examples. As in the above example, there are operators T_j which pair up to get masses, via quadratic superpotential terms. Our conjecture that $R^{(0)}(X) < 5/3$ does not apply to them, because they're not IR free massless fields.

3.3. Misleading anomaly matching examples

In [9] it was pointed out that highly non-trivial anomaly matching, suggesting confined IR free fields, can sometimes be a misleading fluke. The example of [9] was $SO(N)$ with a matter field S in the ⊓⊓. The conserved, anomaly free R-charge is $R(S) = 4/(N+2)$, and the $\mathrm{Tr}U(1)_R$ and $\mathrm{Tr}U(1)_R^3$ 't Hooft anomalies of the asymptotically free UV fields were

shown to match with those of the composite operators $\mathcal{O}_n = \mathrm{Tr} S^n$, $n = 2 \ldots N$, suggesting that the IR theory confines to a free field theory of these composites. But by deforming the theory, both by moving away from the origin of the moduli space of vacua, and by adding superpotential terms, it was shown in [9] that the non-trivial anomaly matching was misleading: the theory at the origin can be shown to instead be an interacting conformal field theory. Here we note that our conjectured condition for accidental symmetries and our diagnostic for confinement, applied to this case, is again successful.

The operators $\mathcal{O}_n$ have R-charge $R_n^{(0)} = R(\mathcal{O}_n) = 4n/(N+2)$, for $n = 2 \ldots N$. Note that all $n \leq (N+2)/6$ have $R_n^{(0)} \leq 2/3$, naively below the unitarity bound, so these operators must become free fields in any case. The issue is what happens with the operators for $n > (N+2)/6$. In the (incorrect) "confining" scenario, the $\mathcal{O}_n$ are IR free fields for all $n = 2 \ldots N$. In the "conformal" scenario, at least some of the operators with $n > (N+2)/6$ remain interacting – to be concrete, we'll suppose that all the operators with $n > (N+2)/6$ remain interacting. Note that for all $n > 5(N+2)/12$, the operators $\mathcal{O}_n$ have $R^{(0)}(X) > 5/3$, above our conjectural upper bound for an operator to become a free field. Let us now compare the conformal anomaly in the two scenarios. Without accounting for any accidental symmetries, the conformal anomaly obtained from (2.1) would be

$$a^{(0)} = \frac{3}{32} \left[3 \frac{(N-1)(5N^2 - 4N + 4)}{(N+2)^2} - (N-1) \right]. \tag{3.16}$$

Now we need to add the accidental symmetry corrections (2.4), to obtain

$$a_{IR}^{interacting} = a^{(0)} + \sum_{n=2}^{(N+2)/6} \frac{1}{96}(2 - 3R_n^{(0)})^2(5 - 3R_n^{(0)}) \approx a^{(0)} + \frac{N}{96}$$

$$a_{IR}^{free} = a^{(0)} + \sum_{n=2}^{N} \frac{1}{96}(2 - 3R_n^{(0)})^2(5 - 3R_n^{(0)}) = \frac{1}{48}(N - 1). \tag{3.17}$$

In the last expression for $a_{IR}^{interacting}$ we simplified by considering $N \gg 1$, using (3.13), and dropping terms which are sub-leading in $1/N$. The last expression for a_{IR}^{free} can also be obtained immediately by applying (2.1) directly to the spectrum of operators $\mathcal{O}_n$, with the free field R-charges $R(\mathcal{O}_n) = 2/3$. Comparing the expressions in (3.17), we here find

$$a_{IR}^{interacting} > a_{IR}^{free}, \tag{3.18}$$

for all N; e.g. for $N \gg 1$, we have $a_{IR}^{interacting} \approx 127N/96$ and $a_{IR}^{free} \approx 2N/96$. So our proposed diagnostic will predict that the IR conformal scenario is favored over the IR confining scenario, in agreement with what was argued to be the correct answer in [9].

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The above example was referred to as $T10$ in the classification table of [28]. The theory T7 in the classification of [28] is the theory of [8], which will be separately discussed in the following subsection. All of the other examples of [28] with $\mu > \mu_{adj}$, i.e. their T8-T11, can also be argued to have misleading anomaly matchings, much as in [9]. These other theories are, T8: $SU(8)$ with matter in the 4-index antisymmetric tensor representation (the **70**); T9: $Sp(4)$ with matter in the 4-index antisymmetric tensor (the **42**); T11: $SO(16)$ with matter in the spinor (the **128**). As pointed out in [28], the dynamics of all of the theories T8-T11 can be related to that of $SO(N)$ with 2-index symmetric tensor, so the analysis of [9] shows that all have interacting conformal, rather than confining phase, at the origin of the moduli space of vacua. For all of these theories, we find that $a_{IR}^{interacting} > a_{IR}^{free}$, so our diagnostic passes the test of predicting the correct IR phase for all of these cases.

Again, the reason why a_{IR}^{free} is less than $a_{IR}^{interacting}$ is because the (incorrect) confining scenario entails accidental symmetries for gauge invariant operators X that would otherwise have $R^{(0)}(X) > 5/3$. E.g. for theory T11, $SO(16)$ with matter S in the spinor, the anomaly free R symmetry is $R(S) = 1 - (\mu/\mu_{adj}) = 1/8$ and the basis of gauge invariants are ([28], table II) $O_n = S^n$ for n =2, 8, 12, 14, 18, 20, 24, 30. The $n = 2$ operator must be free in any case, because of the unitarity bound. The operators with $n = 14$ and higher have $R^{(0)} > 5/3$, and our stronger conjecture is that none of these can be IR free.

3.4. $SU(2)$ with Q in the $I = 3/2$ representation

This theory was discussed in [8]. The anomaly free $U(1)_R$ symmetry has $R(Q) = 3/5$. The basic gauge invariant composite that can be formed is $X = Q^4$, with $R^{(0)}(X) = 12/5$. The TrR^3 and TrR 't Hooft anomalies of the UV fields happen to match with those of X: Tr$R = 3 + 4(3/5 - 1) = (12/5 - 1)$ and Tr$R^3 = 3 + 4(3/5 - 1)^3 = (12/5 - 1)^3$. This matching, which suggests that the IR theory at the origin of the moduli space of vacua consists of just the confined, IR free field X, "seems too miraculous to be a coincidence". But as noted in [8], there is also the possibility that the theory at the origin is instead an interacting conformal field theory. Let us discuss the two possibilities:

In the interacting, conformal field theory scenario, the value of a is, using (2.1),

$$a_{IR}^{interacting} = \frac{3}{32}\left(3\frac{343}{125} - \frac{7}{5}\right) = 0.6405 \qquad (3.19)$$

(assuming that there are no overlooked accidental symmetries). Now consider deforming the theory by the superpotential $W_{tree} = \lambda X$. Because $R^{(0)}(X) > 2$, this superpotential

is irrelevant in the IR, $W_{tree} \to 0$, and the deformed theory simply flows back again to the same interacting conformal field theory in the IR – adding W_{tree} does nothing.

In the IR free, confining scenario, the value of a is that of the massless free field X:

$$a_{IR}^{free} = \frac{3}{32} \left(3(\frac{2}{3} - 1)^3 - (\frac{2}{3} - 1) \right) = \frac{1}{48}. \tag{3.20}$$

In this case, deforming the theory by $W_{tree} = \lambda X$ is a relevant deformation, and it would break supersymmetry [8], dynamically because of the confinement (classically, there'd be a supersymmetric vacuum at $Q = 0$). This is the scenario that was hoped for in [8].

The correct IR phase remains an unsettled question. The difficulty, as compared with other supersymmetric theories, is that there are so few ways to deform the theory.

We note that

$$a_{IR}^{interacting} > a_{IR}^{free}. \tag{3.21}$$

The extra $U(1)_X$ accidental symmetry of the IR free confined scenario leads to a reduction of a; this is because $R^{(0)}(X) = 12/5$, which exceeds our conjectured upper bound $R^{(0)}(X) \leq 5/3$ for fields that can become free in the IR. So the conjectured diagnostic of this paper favors the interacting conformal phase scenario.

Acknowledgments

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