

A Chiral Perturbation Expansion for Gravity

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Abstract

A formulation of Einstein gravity, analogous to that for gauge theory arising from the Chalmers-Siegel action, leads to a perturbation theory about an asymmetric weak coupling limit that treats positive and negative helicities differently. We find power counting rules for amplitudes that suggest the theory could find a natural interpretation in terms of a twistor-string theory for gravity with amplitudes supported on holomorphic curves in twistor space.

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Tree-level MHV amplitudes for (super)Yang-Mills theory [1, 2] have an elegant formulation in twistor space [3], and Witten considered the extension of this to general amplitudes in [4], where it was conjectured that amplitudes are non-zero only if all the external particles in a scattering process are represented by points in twistor space that lie on an algebraic curve of degree d given by

$$d = q - 1 + l, \tag{1}$$

where q is the number of negative helicity particles and l is the number of loops. This can be understood as resulting from an underlying twistor string theory [4, 5, 6] and twistor string theory also leads to conformal (super)gravity, where similar results apply [7]. There has since been great progress in understanding general super-Yang-Mills amplitudes in twistor space; see e.g. [8] and references therein.

The twistor strings of [4, 5, 6] have the problem that conformal supergravity is inextricably mixed in with the gauge theory, so that conformal supergravity modes propagate on internal lines in gauge theory loop amplitudes and there appears to be no decoupling limit giving pure super-Yang-Mills amplitudes. A twistor string that gave Einstein supergravity coupled to super Yang-Mills would be much more useful, and might have a limit in which the gravity could be decoupled.

It is known that MHV amplitudes for Einstein (super) gravity [9] also have an elegant formulation in twistor space [4, 10, 11]. Our purpose here is to seek further evidence that (super)gravity amplitudes could arise from a twistor string theory. An interesting way of understanding (1) in gauge theory [4] is that it follows naturally from the perturbation theory of the Chalmers-Siegel chiral formulation of Yang-Mills theory [12], in which positive and negative helicities are treated very differently. Moreover, the Chalmers-Siegel formulation is precisely the form of the gauge theory that arises from the perturbative twistor string of ref. [4]. We will investigate here chiral formulations of gravity, and show that perturbation theory about them again leads to the relation (1), suggesting that such a chiral formulation of gravity might arise from a twistor string.

The action for Yang-Mills can be written as

$$\int d^4x \text{Tr} \left(G^{\mu\nu} F_{\mu\nu} - \frac{\epsilon}{2} G^{\mu\nu} G_{\mu\nu} \right), \tag{2}$$

where $F = dA + A \wedge A$ is the Yang-Mills field strength and $G = *G$ is a self-dual auxiliary 2-form taking values in the gauge algebra. Eliminating the auxiliary field $G_{\mu\nu}$ from (2) gives the Yang-Mills action

$$\frac{1}{2\epsilon} \int d^4x \text{Tr} \left(F_{\mu\nu}^{(+)} F^{(+)\mu\nu} \right), \tag{3}$$

where $F^{(\pm)}$ are the self-dual and anti-self dual parts of the field strength: $F^{(\pm)} = \frac{1}{2}(F \pm *F)$ in signatures (2, 2) and (4, 0), or $F^{(\pm)} = \frac{1}{2}(F \pm i *F)$ in Lorentzian signature. Here we will

present formulae for signatures (2, 2) and (4, 0) so that all fields are real; the generalisation to signature (3, 1) is straightforward, but involves complex actions. The action (2) is

$$\frac{1}{4g_{YM}^2} \int \text{Tr}(F \wedge *F) + \frac{1}{4g_{YM}^2} \int \text{Tr}(F \wedge F), \quad (4)$$

where $g_{YM}^2 = \epsilon/2$. The first term is the usual Yang-Mills action, and the second is a topological term proportional to the 2nd Chern number that does not affect the equations of motion or perturbation theory.

The weak coupling limit given by setting $\epsilon = 0$ in (2) gives Siegel's chiral theory [13] (see also [14]) with action

$$\int \text{Tr}(G \wedge F) = \int d^4x \text{Tr}(G \wedge F^{(+)}). \quad (5)$$

The field G is a Lagrange multiplier field whose variation gives the constraint

$$F^{(+)}(A) = 0, \quad (6)$$

implying $F = F^{(-)}$, so that the field strength is anti-self-dual. The field equation obtained by varying A is

$$D^\mu G_{\mu\nu} = 0, \quad (7)$$

where $D_\mu = \partial_\mu - A_\mu$ is the gauge covariant derivative. Eq. (7) is of the same form as the Yang-Mills equation $D^\mu F_{\mu\nu} = 0$. The theory describes a helicity +1 particle represented by the Yang-Mills field A with field strength satisfying (6) (so that $F = F^{(-)}$ satisfies $D^\mu F_{\mu\nu}^{(-)} = 0$) and a helicity -1 particle represented by the independent field $G^{(+)}$ satisfying $D^\mu G_{\mu\nu}^{(+)} = 0$. The linearized spectrum is the same for (5) and (4), but the interactions are different: the action (5) has an AGG term, describing a vertex of three fields with helicities $++-$, but, in contrast to Yang-Mills theory, it has no $--+$ vertex.

The theory with action (5) has the same spectrum as Yang-Mills theory, viz. particles of helicities +1 (represented by A) and -1 (represented by G), but differs in the interactions, and it is a non-trivial weak coupling limit of the standard theory written in the form (2). Perturbation theory in ϵ based on the action (2) is an expansion about Siegel's theory (5) and treats positive and negative helicity gluons rather differently.

It is useful to attribute to the independent fields A and G the weights $w[A] = 0$ and $w[G] = -1$ under a $U(1)$ transformation, related to the 'anomalous' $U(1)$ R-symmetry S in the $N = 4$ supersymmetric extension of action (4) [4], with $S = 4w$. The Yang-Mills action (4) then has weight $w = 0$ while the Siegel action (5) has weight $w = -1$ and the second term in (2) has weight $w = -2$. In [4], the $w = -1$ term (5) was interpreted as the transform to space-time of holomorphic Chern-Simons theory on twistor space, while

the $w = -2$ interaction term was related to nonperturbative D-instanton contributions in twistor-string theory.

Consider the perturbation theory in ϵ for the action (2), which was analysed in [4]. If one attributes weights $w = 1$ to ϵ and $w = -1$ to the Planck constant \hbar , then the action rescaled by $1/\hbar$ has weight $w = 0$. The generating functional of scattering matrix elements at l -loops must be a sum of terms of the form

$$\hbar^{l-1} f(A) \epsilon^d G^q \quad (8)$$

for some function f , and for this to have total weight zero it is necessary that the relation (1) holds for each term in the effective action [4]. The power q of G is the number of negative helicity gluons in an l -loop scattering process, while the power d of ϵ is the instanton number, given by the degree of the holomorphic curve in twistor space.

We now turn to gravity, formulated in terms of a vierbein $e^a{}_\mu$ and spin-connection $\omega_\mu{}^{bc}$, with corresponding one-forms e^a, ω^{bc} . The torsion and curvature 2-forms are given by

$$T^a = de^a + \omega^a{}_b \wedge e^b \quad (9)$$

and

$$R^a{}_b(\omega) = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b. \quad (10)$$

In a second order formalism, one imposes the constraint

$$T^a = 0, \quad (11)$$

which determines the spin-connection in terms of the vierbein:

$$\omega_{\mu ab} = \Omega_{\mu ab}(e). \quad (12)$$

Here $\Omega_{\mu ab}(e)$ is the usual expression for the Lorentz connection in terms of the vierbein,

$$\Omega_\mu{}^{ab}(e) \equiv e^{\nu a} \partial_{[\mu} e_{\nu]}{}^b - e^{\nu b} \partial_{[\mu} e_{\nu]}{}^a - e^{\rho a} e^{\sigma b} \partial_{[\rho} e_{\sigma]}{}_c e_\mu{}^c. \quad (13)$$

The Einstein-Hilbert action is

$$\frac{1}{4\kappa^2} \int e^a \wedge e^b \wedge R^{cd}(\omega) \varepsilon_{abcd}. \quad (14)$$

The same action can be used in the first order formalism, in which the torsion is unconstrained and the vierbein $e_\mu{}^a$ and the connection $\omega_\mu{}^{ab}$ are treated as independent variables. The field equation obtained by varying ω is (11), which implies that the Lorentz connection is the Levi-Civita connection (12). The vielbein field equation then gives the Einstein equation.

In Euclidean signature $(4, 0)$, the spin group factorises as $Spin(4) = SU(2) \times SU(2)$ while in split signature it factorises as $Spin(2, 2) = SU(1, 1) \times SU(1, 1)$. The spin-connection decomposes into the self-dual piece $\omega^{(+ab)}$ and the anti-self-dual piece $\omega^{(-ab)}$,

$$\omega_{bc}^{(\pm)} \equiv \frac{1}{2} \left(\omega_{bc} \pm \frac{1}{2} \varepsilon_{bc}{}^{de} \omega_{de} \right), \quad (15)$$

which are the independent gauge fields for the two factors of the spin group.

The curvature 2-form can also be split into self-dual and anti-self-dual pieces

$$R_{bc}^{(\pm)} \equiv \frac{1}{2} \left(R_{bc} \pm \frac{1}{2} \varepsilon_{bc}{}^{de} R_{de} \right), \quad (16)$$

and it is easily seen that $R^{(+ab)}$ depends only on $\omega^{(+)}$ while $R^{(-ab)}$ depends only on $\omega^{(-)}$, with

$$R^{(\pm)a}{}_{b}(\omega) = d\omega^{(\pm)a}{}_{b} + \omega^{(\pm)a}{}_{c} \wedge \omega^{(\pm)c}{}_{b}. \quad (17)$$

In 2-component spinor notation, where α, β transform under the first $SU(2)$ or $SU(1, 1)$ factor and $\dot{\alpha}, \dot{\beta}$ transform under the second, $\omega^{(+ab)}$ becomes $\omega^{\alpha\beta}$ and $\omega^{(-ab)}$ becomes $\omega^{\dot{\alpha}\dot{\beta}}$.

An equivalent form of the Einstein-Hilbert action (14) is given using $R^{(+)}$ instead of R by

$$\frac{1}{2\kappa^2} \int e^a \wedge e^b \wedge R_{ab}^{(+)}(\omega). \quad (18)$$

This gives the action (14) plus the topological term

$$\frac{1}{2\kappa^2} \int e^a \wedge e^b \wedge R_{ab}(\omega). \quad (19)$$

Using (9) and (10) this can be written as

$$\frac{1}{2\kappa^2} \int d(T^a \wedge e_a), \quad (20)$$

which vanishes in the second order formalism in which one sets $T^a = 0$, and in the first order formalism is a total derivative that does not contribute to the field equations or Feynman diagrams. As $R^{(+)}$ depends only on $\omega^{(+)}$, the action (18) is independent of $\omega^{(-)}$ and depends only on the vierbein and the self-dual spin-connection. Moreover, the first order action is polynomial in these variables.

The form (18) of the action has been used as a covariant basis for the reformulation of general relativity in terms of Ashtekar variables, and can be rewritten in two-component spinor notation as [17, 18]

$$\int \Sigma^{\alpha\beta} \wedge R_{\alpha\beta} - \frac{1}{2} \psi_{\alpha\beta\gamma\delta} \Sigma^{\alpha\beta} \wedge \Sigma^{\gamma\delta} \quad (21)$$

where the curvature 2-form $R_{\alpha\beta}$ of $\omega^{\alpha\beta}$ is given in eq. (17) and Σ is a self-dual 2-form acting as a Lagrange multiplier. The totally symmetric Lagrange multiplier field $\psi_{\alpha\beta\gamma\delta}$ imposes the constraint

$$\Sigma^{(\alpha\beta} \wedge \Sigma^{\gamma\delta)} = 0 \quad (22)$$

which implies that

$$\Sigma^{\alpha\beta} = e^{\alpha}_{\dot{\alpha}} \wedge e^{\beta\dot{\alpha}} \quad (23)$$

for some tetrad $e^{\alpha\dot{\alpha}}$. Solving for $\Sigma^{\alpha\beta}$ as in (23) and substituting in (21) yields (18).

It is remarkable that one only needs the self-dual part of the spin-connection in order to formulate gravity. The torsion constructed from $e, \omega^{(+)}$ is (setting $\omega^{(-)} = 0$ in (9))

$$\tilde{T}^a = de^a + \omega^{(+)}{}^a{}_b \wedge e^b. \quad (24)$$

If one imposes the constraint $\tilde{T}^a = 0$, one obtains

$$\omega^{(+)}{}^{ab} = \Omega^{(+)}{}^{ab}(e) \quad (25)$$

and

$$\Omega^{(-)}{}^{ab}(e) = 0. \quad (26)$$

Now (26) implies in turn that

$$R^{(-)}{}^{ab}(e) = 0, \quad (27)$$

where $R^{(-)}{}^{ab}(e)$ is the anti-self-dual part of the curvature of the connection $\Omega(e)$. Then the Riemann curvature constructed from the vierbein is self-dual and hence Ricci-flat, so that the torsion constraint $\tilde{T}^a = 0$ imposes the field equations of self-dual gravity as well as solving for the spin-connection in terms of the vierbein [19].

Siegel [19] gave a remarkable asymmetric action for gravity that is analogous to the asymmetric gauge theory action (5) by introducing a Lagrange multiplier field to impose the constraint $\tilde{T} = 0$. In the second order formalism, $\omega^{(+)}{}^{ab}$ is given in terms of e by (25) and the remaining part of $\tilde{T}^a = 0$ is imposed by a Lagrange multiplier $\sigma_{\mu}^{(-)ab}$ which is anti-self-dual, or in spinor notation $\sigma_{\mu}^{\dot{\alpha}\beta}$. This has the same index structure as the missing anti-self-dual spin-connection. Siegel's action can be written as

$$\int \sigma^{\dot{\alpha}\beta} \wedge \tilde{T}^{\alpha}_{\dot{\alpha}} \wedge e_{\alpha\dot{\beta}}. \quad (28)$$

Varying σ imposes the self-dual gravity equation (26) so that e represents a graviton of helicity -2 . Varying e gives

$$d\sigma^{\dot{\alpha}\beta} \wedge e_{\alpha\dot{\beta}} = 0, \quad (29)$$

so that the Ricci tensor constructed from the linearised curvature $d\sigma$ for an anti-self-dual connection σ vanishes, and the Lagrange multiplier field represents a graviton of helicity $+2$. This action then represents particles of helicity ± 2 , as in Einstein's theory, but the interactions are different for the two helicities, and in particular the theory is linear in σ .

There is also a first-order form of this theory, in which $\omega^{(+)\,ab}$ is an independent field and a Lagrange multiplier is introduced to impose the full constraint $\tilde{T} = 0$ [19], which in turn implies the field equations [19]. Siegel also generalised (28) to give an asymmetric form of $N = 8$ supergravity, with Lagrange multipliers imposing torsion constraints of the supergravity theory [19].

Siegel's asymmetric theory of gravity can be put in a different form that arises as a weak-coupling limit of the Einstein theory, and gives a chiral perturbation theory of gravity [15, 16] similar to that arising from the Yang-Mills action (2). The gravity action (18) depends on the vierbein and $\omega^{(+)}$ only, and $\omega^{(-)}$ decouples completely: we can write it in the form

$$\frac{1}{2} \int e^a \wedge e^b \wedge (d\omega_{ab} + \kappa^2 \omega_{ac} \wedge \omega^c_b), \quad (30)$$

where from now on we omit the superscript $(+)$ so that $\omega \equiv \omega^{(+)}$ and we have rescaled the connection by the gravitational coupling κ^2 . Varying (30) independently with respect to ω^a_b and e^a_μ , we obtain (25) giving the connection in terms of the vierbein, and the Einstein equation

$$e^a \wedge (d\omega_{ab} + \kappa^2 \omega_{ac} \wedge \omega^c_b) = 0. \quad (31)$$

Now taking the limit $\kappa \rightarrow 0$ in (30) yields a weak-coupling limit of gravity with action

$$\frac{1}{2} \int e^a \wedge e^b \wedge d\omega_{ab}, \quad (32)$$

which can be rewritten using eq. (13) as

$$- \int e^a \wedge e^b \wedge \omega_{ac} \wedge \Omega^c_b = - \int e^a \wedge e^b \wedge \omega_{ac} \wedge \Omega^{(+)\,c}_b \quad (33)$$

where $\Omega^{(+)} = \Omega^{(+)}(e)$ is the self-dual part of the connection (13). This is an action for two independent fields, the vierbein e^a_μ and the self-dual connection ω^{ac} ; the latter now plays the role of a Lagrange multiplier field. Note that the self-duality of ω^{ac} implies that only the self-dual part $\Omega^{(+)}$ of $\Omega(e)$ occurs in the action. The field equation from varying the Lagrange multiplier field ω^{ac} sets the self-dual part of $\Omega(e)$ to zero,

$$\Omega^{(+)\,a}_b(e) = 0. \quad (34)$$

This implies that the self-dual part of the curvature constructed from the Levi-Civita connection $\Omega(e)$ vanishes

$$R_{\mu\nu}{}^{ab}(\Omega^{(+)}) = 0, \quad (35)$$

so that the vierbein gives a metric with anti-self-dual Riemann curvature. The field equation for the vierbein gives

$$e_b \wedge d\omega^{ab} = 0. \quad (36)$$

Comparing with (31), this can be seen to be a version of the Einstein equation linearised around the anti-self-dual background spacetime described by the tetrad e^a , where the linearised graviton field is the self-dual connection ω^{ac} .

The fact that ω^{ab} and $\Omega^{(-)}(e)$ are respectively self-dual and anti self-dual means that they describe particles of opposite helicity: e describes a particle of helicity +2 and ω describes a particle of helicity -2. The linearized spectrum is the same for (32) and (14), but the interactions differ as (32) has no $- - +$ vertex. The asymmetric theory (32) is equivalent to the Siegel theory (but with opposite conventions to those used earlier; in (28) e represents a negative helicity graviton, while in (32) it is a positive helicity one).

Now the form of the action (30) has the weak-coupling limit (32), and one can consider perturbation theory in κ^2 about this weak-coupling limit, in complete analogy with that of the gauge theory (2). As in that case, it is useful to attribute to the independent fields e^a and ω^{ab} the weights $w[e] = 0$ and $w[\omega] = -1$ under a $U(1)$ transformation, corresponding to the ‘anomalous’ $U(1)$ R-symmetry S in the $N = 8$ supersymmetric extension of action (28) [19], with $S = 8w$. The chiral action (18) has weight $w = -1$ and the second term in (30) has weight $w = -2$. It is tempting to conjecture that the $S = -16$ interaction term is related to nonperturbative contributions in a new twistor-string theory for Siegel’s truncation of $N = 8$ supergravity. A similar conjecture was made in [10] based on an analysis of maximally helicity violating scattering amplitudes for gravitons.

Consider the perturbation theory in κ^2 for the theory defined by action (30). If one attributes weights $w = 1$ to κ^2 and $w = -1$ to the Planck constant \hbar , then the action rescaled by $1/\hbar$ has weight $w = 0$. The generating functional of scattering matrix elements at l -loops must be a sum of terms of the form

$$\hbar^{l-1} \tilde{f}(e) \kappa^{2d} \omega^q \quad (37)$$

for some function \tilde{f} , and for this to have total weight zero it is again necessary that the relation (1) holds for each term in the effective action. The power q of ω is the number of negative helicity gravitons in an l -loop scattering process in the theory defined by (30). If the theory has a twistor string origin similar to that of [4], then the power d of κ^2 might arise as an instanton number, and the scattering would have support on curves in twistor space characterised by the integer d .

This formulation of gravity extends to one of $N = 8$ supergravity in which the Einstein term is written in the form (30) and the vector field kinetic terms take the form (2). In the weak coupling limit, it gives Siegel's chiral $N = 8$ supergravity [19].

We have seen that the formulation of gauge theory with action (2) and that of gravity with action (30) have many similarities: both have a non-trivial asymmetric weak-coupling limit and both have a perturbation theory that leads to the relation (1). For the gauge theory, the action (2) arises from a twistor string theory with the first term arising from the perturbative theory and the second term from instanton corrections. The constraint (1) then implies that amplitudes are supported on holomorphic curves of degree d in twistor space. It is natural to conjecture that the gravity action (30), or its $N = 8$ supergravity generalisation, should also have an elegant twistor theory origin, and that the formula (1) has a similar twistor space interpretation. We will discuss the twistor formulation of this theory elsewhere.

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