Twistor Strings, Gauge Theory and Gravity

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INTRODUCTION

Within a couple of years from now, the Large Hadron Collider (LHC) will begin operation at CERN, providing proton-proton collisions at 14 TeV center of mass (COM) energy and opening a new window into physics at the shortest distance scales. The luminosity should be $\sim 10-100$ times greater than at Fermilab's Tevatron and the combined rise in energy and luminosity will hugely increase the production of particles with masses in the range 10^2 to 10^3 GeV, including electroweak vector bosons, top quarks, Higgs bosons, and perhaps new particles representing physics beyond the Standard Model (SM). In this context, a detailed quantitative understanding of both the new physics signals and the SM backgrounds is required.

The SM backgrounds for e. g. processes that may contain several jets and (perhaps) a few electroweak bosons are very complex, but a simplifying feature is that the masses of the observed final-state particles in these reactions are generally negligibly small (except for the cases of the W, Z, or top quark, which however immediately decay to essentially massless quarks or leptons). So if we include the decay processes in the description of the event, every final-state particle is approximately massless. We can also (usually) neglect the masses of the colliding partons (the quarks and gluons). Thus the backgrounds (and many signals) require a detailed understanding of scattering amplitudes for many ultrarelativistic (massless) particles, in particular of the quarks and gluons of quantum chromodynamics (QCD).

In principle, Feynman rules are all we need to evaluate the tree and loop amplitudes. In practice, although Feynman rules are very general and apply to any local quantum field theory, the standard method of drawing up all diagrams, computing them using the Feynman rules and then summing all the terms becomes extremely inefficient and cumbersome as the number of external legs grows. The development of new analytic and computational methods in the perturbative approach to Yang-Mills (YM) theories is thus of great and imminent experimental importance: without clear theoretical predictions from perturbative QCD, there would be difficulties in interpreting LHC data. Some new and exciting progress has recently been made in this direction using a variety of field theoretic and string theory-inspired techniques which provide very efficient and powerful alternatives to the usual Feynman diagram expansion. These gauge theory breakthroughs and many others were directly inspired by the recent development of *twistor string theory*, which is a new form of string theory defined on supertwistor space.

The main reasons for the inefficiency of the Feynman diagram expansion are:

- Too many diagrams: many diagrams are related by gauge invariance.
- Too many terms in each diagram: nonabelian gauge boson self-interactions are complicated.
- Too many kinematic variables (allowing the construction of arbitrarily complicated expressions).

Consequently, intermediate expressions tend to be vastly more complicated than the final results, when the latter are represented in an appropriate way. In order to fix ideas, let me give two examples of this:

1. In QCD, the tree-level amplitudes for the scattering of n gluons all vanish if the helicities of the gluons (considered as outgoing particles) are either a) all the same b) all the same, except for one of opposite helicity. Using parity, we can take the bulk of the gluons to have positive

helicity, and write this vanishing relation as

$$A_n^{tree}(1^{\pm},2^+,3^+,\ldots,n^+) = 0.$$

This vanishing is somewhat mysterious from the point of view of Feynman diagrams.

2. The first sequence of nonvanishing tree amplitudes has two gluons with negative helicity, labelled by j and l, say, and the rest of positive helicity. This sequence of maximally helicity-violating (MHV) amplitudes has the exceedingly simple Parke-Taylor (PT) form [1]:

$$\begin{aligned} A_n^{MHV,jl} &= A_n^{tree}(1^+,2^+,\ldots,j^-,\ldots,l^-,\ldots,n^+) \\ &= i\frac{\langle jl\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle} \end{aligned}$$

in terms of $SU(2)$ invariant spinor products

$$\langle ij\rangle = \varepsilon^{AB}\lambda_A\tilde{\lambda}_{A'} \qquad A=0,1 \qquad A'=0',1'.$$

Here we have conveniently written the massless momenta $k_i^2=0$ of the gluons as the product of a left-handed spinor with a right-handed one using the 2-spinor notation:

$$k_i^{\mu}(\sigma_{\mu})_{AA'} = (\lambda_i)_A(\tilde{\lambda}_i)_{A'}.$$

The PT expression above gives the piece of the full amplitude in which the n gluons have a definite cyclic ordering; the full amplitude can be built out of permutations of such partial amplitudes. The formula is extremely simple, and in particular it is ‘holomorphic’ in the spinor variables, a fact which is very mysterious from the point of view of Feynman diagrams.

TWISTOR SPACE AND TWISTOR STRINGS

A breakthrough in the interpretation of such properties of YM scattering amplitudes came when Witten transformed the PT amplitude from the traditional momentum-space variables into Penrose’s twistor space [2]. The twistor transform is a kind of Fourier transform, and there are many examples where transforming a problem into the right variables can expose its simplicity. In this case, Witten found a remarkable interpretation of the formula and of its ‘holomorphy’ properties in terms of a particular string theory on a particular supersymmetric (SUSY) version of twistor space.

Witten defined twistor string theory as a topological B-model with target the supermanifold $\mathbb{CP}^{3|4}$, aka $N=4$ projective supertwistor space. What is projective $N=4$ supertwistor space $\mathbb{CP}^{3|4}$? First, consider complexified flat spacetime $\mathbb{C}^4$. Spacetimes of signature (4,0), (3,1) or (2,2) can be obtained by restricting to suitable real slices of the complexification. Complexified twistor space $\mathbb{T}$ is another copy of $\mathbb{C}^4$ with coordinates $Z^{\alpha}=(Z^0,Z^1,Z^2,Z^3)$. Projective twistor space $\mathbb{PT}$ is $\mathbb{CP}^3$ ($Z^{\alpha}\sim\lambda Z^{\alpha}$ for some $\lambda\in\mathbb{C}^*$) and a point Z in $\mathbb{PT}$ has homogeneous coordinates $[Z^0,Z^1,Z^2,Z^3]$. $\mathbb{T}$ can be defined as the representation space of spinors of the complexified conformal group $SL(4,\mathbb{C})$. The Z^{α} transform as a $\mathbf{4}$ of $SL(4,\mathbb{C})$ and decompose into 2-component spinors under the complexified Lorentz group $SL(2,\mathbb{C})\times SL(2,\mathbb{C})\subset SL(4,\mathbb{C})$; we write $Z^{\alpha}=(\omega^A,\pi_{A'})$ with $A=0,1$ and $A'=0',1'$.

Twistor theory has many important and useful applications, including the Ward construction [3] solving the self-dual (SD) YM equations and Penrose’s ‘nonlinear graviton’ construction [4] solving the self-dual Einstein equations. Most stem from the following basic relationship. A point $x^{AA'}\in\mathbb{CM}$ corresponds to a 2-dimensional linear subspace of $\mathbb{T}$ given by the incidence relation

$$\omega^A = x^{AA'}\pi_{A'}$$

and to a projective line (a copy of $\mathbb{CP}^1$) in $\mathbb{PT}$.

The basic correspondence between twistor space and spacetime is:

$\mathbb{PT}$	$\mathbb{CM}$
complex projective line	point
point	alpha-plane
intersection of lines	null separation of points

Complexified Minkowski space $\mathbb{CM}$ can be thought of as the moduli space of complex projective lines, while (projective) twistor space $\mathbb{PT}$ can be thought of as the moduli space of α-planes [5].

One remarkable application of twistor space is the Penrose transform [5]. This identifies solutions to (massless) free field equations for spacetime fields of arbitrary helicity $-\frac{n}{2}$ on a suitable region $U \subset \mathbb{CM}$ with the cohomology group $H^1(\mathbb{PT}(U), \mathscr{O}(n-2))$, where $\mathbb{PT}(U) \subset \mathbb{PT}$ is the corresponding subset of $\mathbb{PT}$. The $N = 4$ SUSY generalisation of the above constructions is more or less straightforward [2, 9].

The B-model on $\mathbb{CP}^{3|4}$ (or any Calabi-Yau target) describes – for open strings – holomorphic bundles and more general sheaves, together with their moduli. Via the Penrose transform, the open strings (for 'space-filling branes' [2]) reproduce the perturbative spectrum of $N = 4$ super YM (SYM). The interactions can also be reproduced, although this is somewhat harder to see. For closed strings the vertex operator for the physical $(0,1)$-form on $\mathbb{CP}^{3|4}$ describes linearised deformations of the complex structure of a suitable region of $\mathbb{CP}^{3|4}$ (e. g. a neighbourhood $\mathbb{PT}_0$ of a projective line in $\mathbb{CP}^{3|4}$). Penrose showed that such complex structure deformations lead to nontrivial deformations of the flat twistor correspondence wherein space-time is deformed from flat space to one with a curved conformal structure with anti-selfdual Weyl curvature. The construction provides a correspondence between curved twistor spaces and conformally anti-selfdual space-times, yielding a general construction of such space-times. This 'nonlinear graviton' construction establishes that space-time $\mathbb{C}\mathscr{M}$ together with its anti-self-dual conformal structure can be reconstructed from the complex structure of curved twistor space $\mathscr{T}$ together with (Υ, Ω), where Υ denotes the Euler homogeneity operator and Ω the holomorphic 3-form, or from $P\mathscr{T}$ and its complex structure. The existence of the correspondence is preserved under small deformations, either of the complex structure on $P\mathscr{T}$, or of the anti-self dual conformal structure on $\mathbb{C}\mathscr{M}$.

In Witten's twistor string theory, the twistor space string field theory action has a term with a Lagrange multiplier imposing $N(J) = 0$, where $N(J)$ is the Nijenhuis tensor of the complex structure J of the deformed region. Via the nonlinear graviton construction, these integrable complex structure deformations of e. g. $\mathbb{PT}_0$ describe solutions of the ASD Weyl equations in spacetime:

$$W_{ABCD} = 0$$

where $W_{\mu\nu\rho\sigma}$ denotes the Weyl tensor with SD and anti-SD (ASD) parts W_{ABCD} and $W_{A'B'C'D'}$. This describes one helicity of conformal gravity, and leads to an action of the form

$$\int d^4x \, \sqrt{g} U^{ABCD} W_{ABCD}$$

with U symmetric in all its indices and of Lorentz spin $(2,0)$. In addition there is a term

$$\int d^4x \, \sqrt{g} U^{ABCD} U_{ABCD}$$

which arises from D-instantons in Witten's topological B-model. Integrating out U gives an action which is equivalent to the conformal gravity action

$$\int d^4x \, \sqrt{g} W^{\mu\nu\rho\sigma} W_{\mu\nu\rho\sigma}$$

(the difference between the two being a topological term). Thus one gets both the spectrum and the interactions of $N = 4$ conformal supergravity (CSUGRA) [6].

Similar results can be derived using an alternative formulation of the twistor string due to Berkovits [7, 8]. The Berkovits model reproduces the correct tree-level SYM amplitudes using ordinary string tree amplitudes as opposed to D-instanton contributions. The construction uses the fact that in split spacetime signature $++--$ there is a 3-real dimensional submanifold $P\mathscr{T}_{\mathbb{R}}$ of complex twistor space, e. g. the standard embedding $\mathbb{RP}^3 \subset \mathbb{CP}^3$ in the flat case, and the data about deformations of the complex structure is encoded in an analytic vector field f on $P\mathscr{T}_{\mathbb{R}}$.

The Berkovits model is a theory of open strings with boundaries on $\mathbb{PT}_{\mathbb{R}}$ and action

$$S = \int_{\Sigma} d^2\sigma \left(Y_I \tilde{\partial} Z^I + \tilde{Y}_J \partial \tilde{Z}^J - \tilde{A} J - A \tilde{J} \right) + S_C.$$

Here the Y's are variables conjugate to the $Z^I = (Z^\alpha, \psi^a)$ (with the ψ^a, $a = 1, \ldots 4$ fermionic coordinates on the 'soul' of $\mathbb{CP}^{3|4}$), A is a $GL(1,\mathbb{C})$ world-sheet gauge field and J is the associated conserved current on the world-sheet Σ. The $GL(1,\mathbb{C})$ invariance insures that the theory is defined on projective twistor space $\mathbb{PT}$ rather than on twistor space $\mathbb{T}$. The world-sheet theory includes a left and right-moving current algebra S_C reminiscent of heterotic string constructions, and this plays a key role both in the quantum consistency of the model and in determining its spacetime symmetries. In particular, it insures cancellation of conformal anomalies: in addition to the usual Virasoro ghost (b,c) system with $c = -26$ there are ghosts (u,v) and $(\tilde{u},\tilde{v})$ for the $GL(1,\mathbb{C})$ symmetry of the action which contribute $c = -2$.

$N = 4$ SYM physical states are created by the dimension one $GL(1,\mathbb{C})$-neutral primary field

$$V_\phi = j_r \phi^r(Z)$$

where $\phi^r(Z)$ are functions of $GL(1)$ charge zero and the j_r with $r = 1, \ldots \dim G$ are the (left-moving) Kač-Moody algebra currents. G becomes a spacetime SYM gauge group. The superfields $\phi(Z)$ are functions of homogeneity degree zero and have an expansion of the form $\phi(Z) = A_1 + \ldots + A_{-1}\psi^4$, which gives an $N = 4$ SYM multiplet via the Penrose transform.

Similarly (I will not give the details here) $N = 4$ CSUGRA physical states are created by an open string vertex operator constructed from a vector field f defined on $\mathbb{PT}_{\mathbb{R}}$, corresponding to deformations of the embedding of $\mathbb{PT}_{\mathbb{R}}$ in $\mathbb{PT}$, together with a vertex operator constructed from a 1-form $g_I dZ^I$ on $\mathscr{T}$:

$$V_f = Y_I f^I(Z) \qquad\qquad V_g = g_I(Z)\partial Z^I.$$

The physical state conditions are

$$\partial_I f^I = 0 \qquad\qquad Z^I g_I = 0$$

and the gauge invariances are

$$\delta f^I = Z^I \Lambda \qquad\qquad \delta g_I = \partial_I \chi.$$

The emergence of CSUGRA is disappointing from the point of view of Witten's original goal of describing pure $N = 4$ SYM: e. g. at tree level the topological B-model computes the amplitudes of $N = 4$ SYM as desired [2], but at loops it computes the amplitudes of $N = 4$ SYM conformally coupled to $N = 4$ CSUGRA [6]. Moreover there does not seem to be any obvious limits which one could take in the B-model or in the Berkovits model in order to decouple the gauge and gravitational sectors. So CSUGRA seems unavoidable in the Witten and Berkovits models, but this theory is generally considered to be inconsistent: it leads to 4th order PDE for the fluctuations of the metric, and thus to a lack of unitarity.

On the other hand, $N = 4$ SYM makes sense without CSUGRA, and it would be desirable to find a perturbative string theory description of it. A twistor string that gave Einstein SUGRA (with 2nd order field equations for the graviton) coupled to SYM would be much more useful, and might have a limit in which the gravity could be decoupled. The spacetime conformal invariance would be broken in such a theory, and in particular this would introduce a dimensionful parameter which could be used to define the decoupling limit.

NONLINEAR GRAVITON AND NEW TWISTOR STRINGS

In fact there is an important variant of the Penrose construction that applies to conformally SD spaces that are also Ricci-flat, so that the full Riemann tensor is self-dual:

$$W_{A'B'C'D'} = 0, \qquad R_{\mu\nu} = 0 \qquad \Rightarrow \qquad R_{A'B'C'D'} = 0.$$

The corresponding twistor spaces $P\mathscr{T}$ then have extra structure. In particular, they have a fibration $P\mathscr{T} \to \mathbb{CP}^1$. The holomorphic 1-form on $\mathbb{CP}^1$ pulls back to give a holomorphic 1-form on $P\mathscr{T}$ which takes the form $I_{\alpha\beta}Z^\alpha dZ^\beta$ in homogeneous coordinates Z^α, for some $I_{\alpha\beta}(Z) = -I_{\beta\alpha}(Z)$ (which are the components of a closed 2-form on the non-projective twistor space $\mathscr{T}$). The dual bi-vector $I^{\alpha\beta} = \frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}I_{\gamma\delta}$ defines a Poisson structure and is called the *infinity twistor*. Choosing a point at infinity, corresponding to such an infinity twistor, breaks the conformal group down to the Poincaré group; e. g. on Minkowski space, the infinity twistor determines the light-cone at infinity in the conformal compactification. A similar situation obtains more generally: the infinity twistor breaks conformal invariance.

Self-dual space-times are obtained by seeking deformations of the complex structure of twistor space as before, but now Ricci-flatness in space-time places further restrictions on the deformations allowed [4, 10]. The vector field f on $\mathbb{RP}^3$ is required to be a Hamiltonian vector field with respect to the infinity twistor:

$$f^\alpha = I^{\alpha\beta}\frac{\partial h}{\partial Z^\beta}$$

for some function h of homogeneity degree 2 on $\mathbb{RP}^3$. In the linearised theory, such a function h corresponds to a positive-helicity graviton in space-time via the Penrose transform, and the non-linear graviton construction gives the generalisation of this to the non-linear theory [4, 10].

The existence of the nonlinear graviton construction suggests seeking twistor strings that are modifications of the Berkovits or the Witten model with explicit dependence on the infinity twistor, such that there are extra constraints on the vertex operators imposing that the deformation of the complex structure be of the form given above. Then the leading term in the action analogous to

$$\int d^4x\,\sqrt{g}U^{ABCD}W_{ABCD}$$

should have a multiplier imposing self-duality, not just conformal self-duality, and further terms quadratic in the multiplier (from instantons in Witten's approach) could then give Einstein gravity.

A formulation of Einstein gravity of just this form was discussed in [12], and an $N = 8$ SUSY version was constructed some time ago by Siegel [13]. In [9], we constructed new twistor string models which appear to give Einstein (super)gravity coupled to (super) Yang-Mills. The new theories are constructed by gauging certain symmetries of the Berkovits twistor string which are defined by the infinity twistor $I^{\alpha\beta}$ (or its appropriate SUSY generalisation I^{IJ}). Their structure is very similar to that of the Berkovits model, but the gauging adds new terms to the BRST operator so that the vertex operators have new constraints and gauge invariances. I will shortly describe the spectra and some of the interactions of 2 classes of theories for which the world-sheet anomalies cancel.

The corresponding target space theories can be expected to be anomalous in general, with the anomalies arising from inconsistencies in the corresponding twistor string model, though the mechanism for this is as yet unknown. This may rule out some of the models we construct, or restrict the choice of gauge group G. The situation is similar to that of the Berkovits and Witten models, which give target space theories that are anomalous in general, with the anomalies canceling only for the 4-dimensional groups $G = U(1)^4$ or $G = SU(2) \times U(1)$.

The 1st class of anomaly-free theories is formulated in $N = 4$ supertwistor space. Gauging a symmetry of the string theory generated by 1 bosonic and 4 fermionic currents gives a theory with the spectrum of $N = 4$ Einstein SUGRA coupled to $N = 4$ SYM with arbitrary gauge group G. Gauging a single bosonic current gives a theory with the spectrum of $N = 8$ Einstein SUGRA, provided the number of $N = 4$ vector multiplets is 6. In the YM sector, the string theory is identical to that of Berkovits, so that it gives the same tree level YM amplitudes. Both gauged theories have the MHV 3-graviton interaction (with 2 positive helicity gravitons and 1 negative helicity one) of Einstein gravity. The other interactions are still being computed, and the results should determine the form of the interacting theories [14].

There are different interacting theories with the spectrum of $N = 4$ Einstein SUGRA (coupled to $N = 4$ SYM): the standard non-chiral Einstein SUGRA and Einstein SUGRA with chiral interactions. For the theory with $\dim G = 6$ and the spectrum of $N = 8$ SUGRA, the interactions could be those of the standard $N = 8$ SUGRA or those of Siegel's chiral $N = 8$ SUGRA.

The 2nd class of string theories is obtained by gauging different numbers of bosonic and fermionic symmetries so that anomalies are cancelled against ghost contributions for strings in twistor spaces with 3 complex bosonic dimensions and any number N of complex fermionic dimensions, corresponding to theories in 4-dimensional spacetime with N supersymmetries. Analysing the spectrum of states arising from ghost-independent vertex operators, one finds:

- For $N = 0$, a theory with the bosonic spectrum of SD gravity together with SD YM and a scalar.
- For $N < 4$, supersymmetric versions of this $N = 0$ SD theory.
- For $N = 4$, a (second) theory with the spectrum of $N = 4$ Einstein SUGRA coupled to $N = 4$ SYM with arbitrary G.

Consistent non-linear interactions are possible classically for the $N = 0$ theory. The field equations are given by a scalar-dependent modification of the equations for SD gravity coupled to SD YM, and the (noncovariant) action is presumably of the Plebanski type. This $N = 0$ theory may be closely related to the interacting theory of SD gravity coupled to SD YM arising from the (Ooguri-Vafa) $N = 2$ world-sheet supersymmetric string [11]. The theories with $N \leq 4$ are supersymmetric extensions of the $N = 0$ theory, and could be consistent at the interacting level if the $N = 0$ theory is. The determination of the precise form of the interactions must await a detailed investigation of the scattering amplitudes in the new twistor string models [14].

An important point here is that if one scales the infinity twistor $I^{IJ} \to RI^{IJ}$, $\varepsilon^{AB} \to R\varepsilon^{AB}$, while keeping f^I, g_I fixed, then $h \to R^{-1}h$ and $\tilde{h} \to R\tilde{h}$. Then the amplitude scales as R^{-1}, so that R^{-1} sets the strength of the gravitational coupling. Thus there is a decoupling limit of gravity in our models, which should be useful in computing YM amplitudes at loops using the twistor string. In the case of the model giving the spectrum (and some of the interactions) of $N = 8$ SUGRA, we hope to use the string theory to calculate $N = 8$ SUGRA loop amplitudes as well as to investigate the conjectured ultra-violet finiteness of this very special SUSY field theory of gravity, and the fascinating and intimate connection of its scattering amplitudes to those of $N = 4$ SYM.

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