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About Time*

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Abstract

We present a novel perspective on the *problem of time* in quantum gravity. Inspired by Einstein's analysis of the concept of time and his equivalence principle, and guided by the geometric structure of quantum theory, we offer new insights into the nature and the origin of time.

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I. Of spacetime and the river

The quest for a theory of quantum gravity is fundamentally an attempt to reconcile two disparate notions of time. Einstein's theory of general relativity teaches us that time is ultimately an illusion while quantum theory tells us that time evolution is an essential part of Nature. Moreover, the failure to resolve the conflict between these competing notions is at the heart of our inability to properly describe the earliest moment of our Universe, the Big Bang. We will spell out in clear terms the nature of the discrepancy between these competing views and lay out a new proposal for a theory of quantum gravity which provides a clear resolution to the problem of time.

Time in quantum mechanics and time in gravity are very different things. In quantum mechanics, time is the fundamental evolution parameter of the underlying unitary group. The Hamiltonian operator of a given system generates translations of the initial state in time. Unlike other conjugate quantities in the theory such as momentum and position, the relation between time and energy, which is the observable associated to the Hamiltonian, is distinguished. Time is not an observable in quantum theory in the sense that there is no associated "clock" operator. In the Schrödinger equation time simply enters as a parameter. This conception of time as a Newtonian construct, global or absolute, in a post-Newtonian theory persists even when we promote quantum mechanics to relativistic quantum field theory. An illustration is the microcausality condition requiring two field operators at the spacetime points X and Y to commute, namely, to have no effect on each other, whenever these points cannot be connected by a light signal.

To get to the notion of time in gravity, we first recall that measurements are also by nature different in quantum field theory and the general theory of relativity [1]. The special theory of relativity and quantum mechanics, as well as their conflation, are formulated in terms of particle trajectories or wavefunctions or fields that are functions (functionals) of positions or momenta. Coordinates, however, must be regarded as auxiliary constructs in the general theory of relativity. In classical gravity, spacetime is simply an arena in which events transpire. The coordinates — including time — that are assigned as labels to the events are not in themselves especially meaningful due to the general covariance of Einstein's theory of gravitation. To better understand the role time plays in general relativity we can consider as an analogy wrapping a birthday present. There are many different kinds of paper with which to wrap a present just as there are many kinds of coordinates that can be used to label events. Time being one of these coordinates is no more fundamental to the events than wrapping paper is to the present. However, as noted above, quantum theory is ill defined without time. If we attempt to discard it like so much used wrapping paper we lose the essence of the theory and our present is destroyed in the process. Although general relativity is formulated as a local classical field theory, it has distinguishing non-local features, as exemplified, by the concept of gravitational energy [2]. No local gauge invariant observables can exist within a theory of gravitation [3]. Rather, the most natural observables are couched in terms of integrals over all of spacetime, for example, $\int \mathrm{d}^d x \sqrt{-g}\, R$ with g and R being, respectively, the determinant of the metric and the scalar curvature. Such observables are manifestly diffeomorphism invariant, but the failure of locality is intrinsic to their definition. The decoupling of scales familiar to local quantum field theories is simply not tenable in gravity. A chief lesson then is that high energy physics mixes inextricably with dynamics at low energy.

Measurements in a classical theory of gravitation are founded on the *relational* properties among spacetime events. Timelike separation of events is measured by mechanical clocks, whereas spacelike separation is determined more indirectly [4]. We might thus prescribe a clock in Greenwich as a standard reference object and consider other events in spacetime relative to it. Then, for example, the curvature invariant $K = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, $R^{\mu\nu\rho\sigma}$ being the Riemann tensor, at an event specified relative to the ticking of the clock in Greenwich is a quasilocal observable in the theory [5].

By contrast within a quantum theory, events cannot themselves be localized to arbitrary precision. Only for high energies does it even make sense to speak of a local region in spacetime where an interaction takes place. This indeterminacy simply follows from the energy-time uncertainty relation. However, the measurements that an observer makes within a quantum theory of gravity are necessarily constrained. Experiments are performed in finite duration and at finite distance scales. The interactions accessed in the laboratory also take place in regions of low spacetime curvature. In local quantum field theory, we operate successfully under the conceit that the light cone is rigid. With respect to the vacuum in flat space, there is an approximate notion of an S-matrix, or scattering amplitude, which is the unitary matrix relating asymptotic initial and final states for an interaction of particles. Even in string theory, computing such amplitudes proceeds through analytic continuation of Lorentzian spacetimes into Euclidean spaces with fixed asymptopia. On cosmological scales, this procedure is of course a cheat. Light cones do tilt, and the causal structure is not static. In general settings, no choice of coordinates will ensure the existence of a global timelike direction throughout all of spacetime. Even if such a coordinate system were to exist, as local observers we do not *a priori* know what the asymptotic behavior of the metric will be at late times. The only data available about spacetime are events in the observer's past light cone. Each observer has a different past light cone consistent with the histories of all the other observers, and the future causal structure is partially inferred from these data.

In a quantum theory of gravity, the spacetime metric is expected to fluctuate. In consequence, notions such as whether the points X and Y are spacelike separated become increasingly blurred as fluctuations amplify. Indeed, Lorentzian metrics exist for almost all pairs of points on a spacetime manifold such that the metric distance $g(X, Y)$ is not spacelike [6]. Clearly the notion of time, even locally, becomes problematic in such quantum gravitational regimes. The failure of microcausality means that the techniques and intuitions of quantum field theory must be dramatically revised in any putative theory of quantum gravity.

Given the overwhelming success of local quantum field theory as the bedrock for our understanding of Nature in all its (albeit non-gravitional) quantum interactions, it is tempting to retain its underlying precepts as we refine and deepen our analysis. But there is an inescapable puzzle here as well. The observational evidence points to a hot Big Bang event some 13.72 ± 0.12 Gyr in the past [7]. While a precise model for the Big Bang in quantum gravity is presently lacking, the Penrose–Hawking theorems point to the existence of an initial singularity [8]. Under conservative assumptions, there exist geodesics that are only finitely extendible into the past. The curvature of spacetime blows up at the singularity, which is the terminus of the geodesics, and general relativity cannot reliably say anything useful about physics anymore since the very notion of geometry breaks down. The origin of the Universe also raises conceptual questions within the quantum theory itself. In particular, if we uphold the canonical definition of the unitary evolution with a globally defined evolution parameter, what happens to that evolution as we approach the singularity? According to quantum theory, the evolution is not supposed to terminate, and yet, from both the physics and the general relativity standpoints, it seemingly does end.

From the above remarks, our standard conceptions of time in quantum theory and in classical general relativity are in extreme tension: time in the quantum theory is an absolute evolution parameter along the real line whereas in general relativity there can be no such one parameter evolution. What, then, may we ask is the nature and the origin of time in quantum gravity?

II. Time is but the stream I go a-fishing in

Developing a theory of quantum gravity which resolves the problem of time is a monumental undertaking. One of the largest obstacles to overcome derives from the fact that the role of time is intimately tied to the underlying structure of quantum theory, a foundation which, as we shall describe, is very rigidly fixed. We will propose a specific way of loosening this structure ever so slightly while still keeping all but one the key ingredients of quantum theory intact. We will discover that following this prescription has surprisingly profound implications for quantum gravity and the problem of time.

In reconciling the identity of time in gravitation and canonical quantum theory one is immediately struck by the marked difference in the most commonly used formalisms of the two theories. General relativity is typically articulated in a geometric language while quantum mechanics is most commonly thought of in terms of an operatorial formalism. Yet when quantum theory is examined in its less familiar geometric form, it mimics general relativity in several essential aspects. In fact, these parallels provide a natural springboard for our extension of the quantum mechanical framework to graft gravity at the root quantum level. The geometric formulation illuminates the nature and rigidity of time in quantum theory and points a way to make time more elastic. As a prelude to such a generalization, we first sum up the key features of geometric quantum mechanics.

In casting quantum mechanics within a geometric framework all the standard features of the theory are present and acquire a geometric interpretation [9]. To begin, each physical configuration of the system, its wavefunction, corresponds to a point in the quantum mechanical phase space. Typically, the phase space is parameterized in terms of coordinates which are positions and momenta. Here these are, respectively, the real and imaginary parts of the Fourier components of the wavefunction in some orthonormal basis. Now a phase space has a natural symplectic structure ω which allows for the definition of a Poisson bracket or commutator and hence provides an arena for Hamiltonian dynamics. Specifically, the phase space of the canonical quantum mechanics is the complex projective space $\mathbb{CP}^n$, visualizable as the set of complex lines through the origin in $\mathbb{C}^{n+1}$, a complex Euclidean space. (The points $(z_1, \ldots, z_{n+1})$ and $(\lambda\, z_1, \ldots, \lambda\, z_{n+1})$, where λ is a non-zero complex number, lie on the same line in $\mathbb{C}^{n+1}$ and are thus identified as a single point in $\mathbb{CP}^n$.) The n denotes the dimension of such a projective Hilbert space of the quantum theory; it is typically infinite. The manifold $\mathbb{CP}^n$ is highly symmetric: it is a compact, homogeneous, isotropic, and simply connected Einstein space with a constant holomorphic sectional curvature $2/\hbar$. In the simplest case of $n = 1$, $\mathbb{CP}^1$ is S^2, the round sphere in three dimensions and is the state space of a two level quantum system or that of the qubit of quantum computation. It is important to note that to have a theory of a quantum system, one key ingredient is still missing: the dynamics. Like classical mechanics, quantum mechanics is not so much a theory as it is a general framework for it does not provide a specific Hamiltonian but asks for one. With a given Hamiltonian, a physical system is thus defined along with its associated relevant variables, observables, etc. As each quantum state $|\psi\rangle$ corresponds to a point in phase space, the metrical distance between two nearby states is determined by the overlap of the corresponding wavefunctions: $g(|\psi\rangle, |\psi'\rangle) \propto (1 - |\langle\psi|\psi'\rangle|^2)$. The metric g on $\mathbb{CP}^n$ is in fact the *unique* Fubini–Study metric, or the Fisher distance from information theory and statistical geometry [10], and captures the Born rule for probabilities. In the classical limit, for a point particle, this metric on the space of quantum states reduces to the spatial metric in spacetime when the configuration space of the quantum system under consideration is the physical space itself. Similarly, the corresponding symplectic form becomes the classical Poisson bracket between the spatial position and momentum of the particle. This reduction shows the path to the emergence of physical space from quantum mechanical degrees of freedom.

The symplectic structure of phase space determines how wavefunctions, or more generally, fields propagate. On $\mathbb{CP}^n$, this time evolution or the Schrödinger equation turns into a geodesic equation with respect to the Fubini–Study metric for a point particle carrying a non-Abelian charge, a disguised form of the Hamiltonian. The manifold $\mathbb{CP}^n$ is curved, and the dynamics are non-linear. The curvature of the space concerns the propagation of the wavefunction. Thus, $\mathbb{CP}^n$ tells the matter particle how to move but, unlike general relativity, matter has yet to tell the state space how to curve. Realizing this backreaction on the phase space will be the key feature of our model of quantum gravity.

In contrast to the classical phase space endowed only with a symplectic structure, the quantum state space is threefold richer in structure. On the complex projective space there are the metric g, the symplectic structure ω, and the almost complex structure J. (J is the mysterious $i = \sqrt{-1}$ that appears in the Schrödinger equation, $J\,\hbar\,\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = H\,|\psi(t)\rangle$.) These are the three pillars upon which geometric quantum mechanics is constructed; any pair of these determines the third. Within this architecture, the superposition principle is tied to viewing $\mathbb{CP}^n$ as a collection of complex lines passing through the origin. Entanglement arises from the Segre embedding of the products of two complex projective spaces into a higher dimensional one. The symplectic structure yields the geometric phase of the wavefunction. The speed with which the system moves through phase space is related to the uncertainty in the energy, ΔE. Specifically we can derive the Aharonov–Anandan relation $\frac{\hbar}{2}\,\mathrm{d}s = \Delta E\,\mathrm{d}t$, which correlates the statistical distance in phase space as given by the Fubini–Study metric to the time in the canonical evolution [9]. This equation is very telling in several respects. Firstly, it says that time is statistical and quantum mechanical in nature. It defines a quantum clock. It indicates that distance in quantum phase space is linked in an exact fashion to time evolution. What the line element $\mathrm{d}s$ on $\mathbb{CP}^n$ measures is the optimal distinguishability of nearby pure states: if the states are hard to resolve experimentally, then they are close to each other in the metrical sense [10]. (This is a key feature of our model in relation to its implications for the nature of the Big Bang which will be discussed subsequently.) Most importantly, recall that the phase space geometry is fixed by the structure of quantum mechanics and while that space is curved, it is constrained to be a constant curvature manifold at every point in the phase space. The rigidity of the geometry, namely the uniqueness and hence universality of the Fubini–Study metric, enforces the constraint that time in quantum mechanics is a global coordinate, that it is absolute. This is very much akin to the view of time in special relativity, whereas in general relativity the metric is dynamical and time is local. It is this crucial observation that captures with precision the tension between time in quantum theory and time in a gravitational context.

In light of the above, we wish to develop a notion of time in quantum theory that is manifestly local and which allows for a quantum phase space with a dynamical metric. However, this is quite a challenge given the rigidity of quantum theory. Let us return to the triadic structure of the metric, the symplectic, and the complex structures. If quantum theory were a photographic tripod, the triadic structure would be its three legs. Without all three, the theory cannot stand, and once we have properly positioned two of the legs, the position and existence of the third is implied. The structure of the tripod is rigid in the sense that if we move one of the legs too far from its position it will collapse. Moreover, adjusting any one of the legs will necessarily require adjustment of the other two in order to keep the tripod upright. It is a delicate balance.

Motivated by physics, we must then proceed to gently shift one of the three defining structures such that we change the postulates of quantum mechanics in a minimal way. The questions we must answer are: which leg to move first, how do we move it, and what its effect will be on the other two legs. For the first question, we recall that we wish to tinker with time in quantum theory and the leg most closely tied to time is the complex structure, due to the role played by i in time evolution. Thus we propose changing this structure from being globally complex to only locally complex or equivalently globally strictly almost complex. This mathematically precise relaxation of the original rigidity will have several remarkable consequences. It allows for an extension of the governing symmetry of quantum theory while ensuring that the symplectic structure and the metric remain compatible with this change. Thus we may extend the unitary group to the general covariance group, the symmetry group of general relativity! We must bear in mind, however, that we are not working not in spacetime but on the space of states, the true arena of quantum dynamics. Thus the quantum phase space $\mathbb{CP}^n$ gives way to a newcomer in physics, the infinite dimensional non-linear Grassmannian $\mathrm{Gr}(\mathbb{C}^{n+1}) = \mathrm{Diff}(\mathbb{C}^{n+1})/\mathrm{Diff}(\mathbb{C}^{n+1}, \mathbb{C}^n \times \{0\})$, in the $n \to \infty$ limit [11, 12]. On the latter complex projective space, which admits scores of metrics, the symplectic structure, the metric structure, and hence also the almost complex structure become dynamical and the global time of quantum theory relaxes to a more provincial, local time. Most importantly, the essential elements of quantum theory are left intact, and the tripod is kept stable.

The requirement of diffeomorphism invariance places stringent constraints on the quantum geometry. We must have a strictly (*i.e.*, a non-integrable) almost complex structure on a necessarily infinite dimensional generalized space of quantum events. It follows that wavefunctions that label the event space, while already unobservable, are in fact irrelevant. They are as meaningless as spacetime coordinates are in general relativity; the physics remains the same with a different choice of labels. Only individual quantum events make sense observationally. At the basic level, in our scheme there are only dynamical correlations between quantum events. Observables are furnished by diffeomorphism invariant quantities in the quantum phase space. A ready example is the line element, $\mathrm{d}s^2$.

The dynamical evolution equation of the statistical metric in our model is formally the same as in the diffeomorphism invariant theory of matter and spacetime geometry:

$$R_{ab} - \frac{1}{2}\, g_{ab}\, R - \Lambda\, g_{ab} = T_{ab} \ , \tag{1}$$

with the energy-momentum tensor on the right hand side being determined by the Hamiltonian of the quantum system under consideration as well as by the holonomic diffeomorphism group gauge fields inherent to the new phase space. Parallelling the case of general relativity, the non-linear geodesic Schrödinger equation is implied by the conservation of the energy-momentum tensor. At last, we have the bootstrapping of matter and state space dynamics: the latter tells matter how to move, the former tells the state space how to curve, and each feeds back on the other.

Given the close parallels between our proposal and general relativity, we may ask whether there is any relation between the phase space of the generalized quantum theory and spacetime. Indeed, it turns out that such a connection exists. Just as in quantum mechanics, our extension also requires the specification of a Hamiltonian with its defining variables. The form of the Hamiltonian may be fixed by the requirement that H should describe a canonical quantum mechanical system whose space of configurations is the physical space and whose dynamics defines a consistent quantum gravity in a flat background as this is the local physics we are after. We know of only one example of a non-perturbative quantum mechanics that satisfies this criterion: *Matrix theory* [13]. By promoting all of the coordinates to matrices, spacetime is expected to emerge from the much less intuitive dynamical quantum phase space.

In closing the presentation of our model, it should be emphasized that the generalization of quantum theory that we have proposed is perfectly sensible. In particular, there is no loss of unitarity in the model. To be more precise, probabilities are conserved in the sense of the more general, dynamical probability measure. In other words, the probabilities sum to one, but the group of transformations of underlying quantum dynamics extends the unitary group and the Born rule is thereby generalized. Much of the structure implied by this model has yet to be worked out in its details, both conceptually and computationally, but given the remarkable properties enumerated above, we believe this proposal offers great promise. The message at the heart of this model is that we must surrender the global notion of time at all scales, while retaining the central role that time evolution plays in Nature.

III. A long while ago the world began

Our Universe started out as an extreme environment, a hot, small, dense state. Thus understanding the Big Bang requires a grasp of physics at the smallest scales in the presence of exceptionally strong gravitational fields. The Universe at that time was by its very nature a crucible in which Einstein's general relativity and quantum theory mix. Moreover, because it is the Universe's initial state, it is inextricably linked to understanding the nature of time.

The initial state of the Universe had a very low entropy meaning that it was a highly ordered state. The entropy of the observed Universe *today* can be estimated by counting the degrees of freedom at the causal horizon:

$$S \simeq \left(\frac{R_H}{\ell_P}\right)^2 \simeq 10^{123} \ , \tag{2}$$

where R_H is the Hubble radius and ℓ_P the Planck length. The number of microstates is then given by Boltzmann's formula $\Omega = e^S \simeq e^{10^{123}}$, and the probability associated with the Big Bang is $P \sim 1/\Omega \simeq e^{-10^{123}}$. The Big Bang therefore appears to be an exceptionally special point in phase space [14], a most unlikely event. That the entropy increases irreversibly with the expansion of the Universe is the so called *cosmological arrow of time*.

Within the framework of our model we examine the possible initial states of the Universe. Refinements in the understanding of the non-linear Grassmannian, the configuration space of the quantum theory, provide a sharp, new insight in this direction. In particular, different metrics may be associated to the space. Since $\mathrm{Gr}(\mathbb{C}^{n+1})$ is the diffeomorphism invariant counterpart of $\mathbb{CP}^n$, the simplest and most natural topological metric to consider is the analog of the Fubini–Study metric. Just such a situation was analyzed by Michor and Mumford [15], who obtained the striking result that the generalized Fubini–Study metric induces on $\mathrm{Gr}(\mathbb{C}^{n+1})$ a vanishing geodesic distance between any two points whatsoever on this space. The fluctuations of the space become wild and the curvature becomes unbounded. This causes the space to wrap tightly on itself.

As an analogy, consider that a shirt may have two buttons on opposite cuffs which do not touch when the shirt is worn but that can be brought into contact when it is folded. In a very similar fashion, the points on the phase space of the quantum theory come into contact with each other. With the choice of the above analog Fubini–Study metric on, the folding of the phase space of the theory becomes so extreme as to bring every point arbitrarily close to every other point. In other words, the space comes to exist, effectively, as a single point.

A point in the phase space of the theory corresponds to a state of the quantum system, which in this case is the entire Universe. The distances in phase space between points corresponds to the ability to distinguish the different quantum states from one another. Thus the result of employing the Fubini–Study metric on the non-linear Grassmannian is that the quantum state space assumes only one possible configuration. The entropy, which is given by the logarithm of the number of possible quantum states, then vanishes. This is precisely the type of configuration that describes the initial state of the Universe.

While the system has entropy $S = 0$, the very high curvatures in $\mathrm{Gr}(\mathbb{C}^{n+1})$ signal a non-equilibrium condition of dynamical instability. The system will flow toward differentiation, which yields, through entropy production, distinguishable states in the quantum phase space. The vanishing geodesic distance can be interpreted as the signature, or order parameter, of a strong fluctuation induced transition from the low-entropy, high-temperature initial state to the high-entropy, low-temperature Universe that we observe and inhabit at late times.

How can this occur? The space $\mathrm{Gr}(\mathbb{C}^{n+1})$ has, in principle, an infinite number of metrics, a subset of which will solve the dynamical equations on the state space. Indeed, there is for instance an infinite one parameter family of non-zero geodesic distance metrics, of which the Fubini–Study metric is a special case [15]. The dynamical evolution, according to the second law of thermodynamics, is toward some higher entropy but stable configuration. During this evolution, spacetime and canonical quantum mechanics will emerge.

Einstein's equations tell us that matter and energy react dynamically to the geometry of spacetime and, conversely, spacetime responds to matter and energy by curving, and the process continues. The model proposed here is a highly non-linear theory that emulates general relativity in its structure. The quantum state space is also governed by non-linear differential equations and will likewise experience back reaction. Because it is a far from equilibrium system, the initial state will decay. The Big Bang is, in fact, very reminiscent of a "freezing by heating" metastable phase [16]. As the configuration space of the Matrix theory is spacetime, when the non-linear Grassmannian evolves away from the initial Big

Bang configuration such that the points of the state space differentiate, the Universe expands. This type of phase transition away from the zero entropy Big Bang state yields the cosmological evolution of the Universe. Thus the Universe is required to undergo cosmological evolution and is naturally equipped with an arrow of time. This cosmological arrow of time is nothing but the flow on the space of metrics on $\mathrm{Gr}(\mathbb{C}^{n+1})$. Because there are more possible configurations of the quantum system when the metric distance between points on the configuration space is non-zero, the Universe consistently evolves to a state of higher entropy.

The geometric structure of the non-linear Grassmannian is remarkable. It addresses the problem of time and also the arrow of time along with a picture of the initial state. We have seen the profound discrepancy between time in quantum mechanics, where it is a necessary parameter for describing the unitary evolution of the system, and time in gravitation, where it is a label with no special meaning. Treating quantum mechanics geometrically and then altering the triadic structure of quantum theory in a consistent way provides a framework in which the two different meanings of time find reconciliation. Because we rely upon Matrix theory as a non-perturbative, non-local formulation of M-theory in a flat background, this supplies a theory of quantum gravity at small scales. On larger scales, where we patch different neighborhoods together as we do with a coordinate atlas on a differential manifold, time assumes its expected character in a diffeomorphism invariant theory of gravity. The metric on $\mathrm{Gr}(\mathbb{C}^{n+1})$ need not be fixed, so we also flow from a configuration space with a single state to a configuration space with e^S states, one for which general relativity approximates the physics in spacetime.

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