Surprising simplicity of N=8 supergravity

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Surprising simplicity of $\mathcal{N} = 8$ supergravity

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Abstract: Gravity amplitudes are via the Kawai-Lewellen-Tye relations intimately linked to products of Yang-Mills amplitudes. Explicitly this show up in computations of $\mathcal{N} = 8$ supergravity where the perturbative expansion and ultraviolet behaviour of this theory is akin to $\mathcal{N} = 4$ super-Yang-Mills at least through three loops. Full persistency to all loop orders would be truly remarkable and imply finiteness of $\mathcal{N} = 8$ supergravity in four dimensions.

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Contents

1. Introduction

Since the discovery of quantum mechanics in the previous century, physicists have been pursuing a construction of a fundamental theory for quantum gravity. Quantum gravitational effects appear to be essential in understanding the physics of very dense matter objects such as the early universe and black holes. However, although the searches for a theory of quantum gravity have been diverse, extensive and many, the fundamental concepts of such a theory are still elusive. General relativity provides us with a very successful theory for gravity which captures the apparent necessary knowledge for a complete treatment of the gravitational attraction and its intimate connection with matter, space and time. However general relativity is incompatible with basic quantum mechanical ideas such as operator space and expectation values. A traditional approach to perturbative gravity through a Lagrangian description is possible although complicated by a divergent ultraviolet behaviour. Progress has however been achieved this way through treating gravity as an effective field theory [1].

For many years the combination of supersymmetry with a Lagrangian description of quantum gravity was considered to be a way out of the troublesome ultraviolet divergent behaviour of such a theory due to the introduction of extra fundamental symmetries. Such theories was termed *supergravity models*. The possibly most famous one is the model of maximal $\mathcal{N}=8$ supergravity [2,3]. However with the advent of superstring theory in the mid-1980ties such models were abandoned due a common belief of unavoidable divergences in their perturbative expansion via power-counting arguments [4] relegating these theories to low-energy effective descriptions of string theory.

In recent years, due to remarkable progress in computational techniques by combining various input from string theory, extended supersymmetry and unitarity, there has been a renewed interest in supergravity models for quantum gravity and it has become clear that

$\mathcal{N}=8$ supergravity has a much better perturbative expansion than power-counting naïvely predicts. Surprisingly the ultraviolet behaviour of $\mathcal{N}=8$ supergravity occurs explicitly to be identical to the one of $\mathcal{N}=4$ super-Yang-Mills to at least three loops [5–12] and very likely six loops [13,14]. If this identical UV-behaviour persists to all orders in perturbation theory then $\mathcal{N}=8$ supergravity will be ultraviolet finite in four dimensions [13].

The massless spectrum of $\mathcal{N}=8$ supergravity can be seen as the tensorial product of two copies of $\mathcal{N}=4$ super-Yang-Mills theories, through the Kawai-Lewellen-Tye relations [15] which are motivated by string theory. In these relations the massless supergravity (closed string) vertex operators are written as the left/right product of Yang-Mills open string vertex operators. One can hence organise $\mathcal{N}=8$ supergravity tree-level amplitudes according to a relation [5,16–19] which we will write schematically in the following way

$$\text{Gravity} \sim (\text{Yang-Mills}) \times (\text{Yang-Mills'})\,. \tag{1.1}$$

This simple relation between a theory of gravity and two gauge theories is observed directly in on-shell S-matrix elements but appear to be rather odd at the level of the Lagrangian and its interactions (This is true even if part of the Lagrangian is rearranged as a product of Yang-Mills types of interactions at the two-derivative level [20,21] or for higher derivative corrections [22].).

2. Supergravity amplitudes

A superficial power counting argument indicates that an L-loop n-graviton amplitude in D-dimension behaves as

$$[\mathfrak{M}_L^{(D)}] \;=\; \text{mass}^{(D-2)L+2}\,. \tag{2.1}$$

This count can be compared to the superficial power counting of the four-gluon amplitude in $\mathcal{N}=4$ super-Yang-Mills which is given by

$$[\mathfrak{A}_{4;L}^{(D)}] \;=\; \text{mass}^{(D-4)L}\,. \tag{2.2}$$

For $\mathcal{N}=4$ super-Yang-Mills in four dimensions we see that the theory is at most logarithmically diverging (since the coupling constant is dimensionless). The extended $\mathcal{N}=4$ supersymmetry guaranties perturbative finiteness [23,24]. Colour ordered amplitudes factorise the dimension four operator F^4 at one-loop and the dimension six operator $\partial^2 F^4$ at higher-loop order and hence satisfy the dimensional analysis

$$[\mathfrak{A}_{4;L}^{(D)}] \;=\; \text{mass}^{(D-4)L-6}\,[\partial^2 F^4]\,. \tag{2.3}$$

This imply that L-loop four-point amplitudes in $\mathcal{N}=4$ super-Yang-Mills are ultraviolet divergent in dimensions

$$D \geq 4 + \frac{6}{L}\,. \tag{2.4}$$

Thus implying perturbative ultraviolet finiteness in $D=4$ dimensions (the negative mass dimension reflect the infrared behaviour of the amplitude).

The difference between the formulæ (2.1) and (2.3) reflects the difference in dimensions of the coupling constant of the two theories. However this superficial power counting misses dramatic simplifications taking place in on-shell amplitudes due to the extended $\mathcal{N} = 8$ supersymmetry [14] and the rôle of (diffeomorphism) gauge invariance [9, 25].

String based methods for constructing higher-loop amplitudes indicate [13,14] that the perturbative behaviour of $\mathcal{N} = 8$ supergravity amplitudes is improved by the factorisation of the dimension eight R^4 operator together with extra powers of derivatives

$$[\mathfrak{M}_L^{(D)}] \;=\; \mathrm{mass}^{(D-2)L-6-2\beta_L}\,[\partial^{2\beta_L} R^4]\,, \tag{2.5}$$

with the $\beta_L = L$ rule

$$\beta_1 = 0; \qquad \beta_L = L \qquad \text{for} \qquad 2 \leq L\,. \tag{2.6}$$

This leads to a superficial ultraviolet behaviour for $\mathcal{N} = 8$ supergravity amplitudes of

$$[\mathfrak{M}_L^{(D)}] \;=\; \mathrm{mass}^{(D-4)L-6}\,[D^{2L} R^4]\,, \tag{2.7}$$

which is *similar* to the ultraviolet behaviour in (2.3) for $\mathcal{N} = 4$ super-Yang-Mills. When the $\beta_L = L$ rule (2.6) is satisfied the $\mathcal{N} = 8$ four-graviton supergravity amplitude has the same critical dimension (2.4) for ultraviolet divergences as $\mathcal{N} = 4$ super-Yang-Mills.

The validity of the $\beta_L = L$ rule to all orders in perturbation theory implies perturbative finiteness of the four-graviton $\mathcal{N} = 8$ supergravity amplitude in four dimensions.

3. $\mathcal{N} = 8$ supergravity as a product of $\mathcal{N} = 4$ Yang-Mills

On-shell recursion relations provide very simple means of constructing $\mathcal{N} = 8$ supergravity tree-level amplitudes from three-point vertices [26, 27]. Gravity three-point vertices are given directly as squares of $\mathcal{N} = 4$ super-Yang-Mills vertices. Thus the resulting massless n-point tree-level amplitudes can be presented in a form involving terms with sums of squares of three-point $\mathcal{N} = 4$ super-Yang-Mills vertices [19].

At the field theory level this amounts to replacing the gauge degree of freedom of the Yang-Mills fields by Lorentz degrees of freedoms as follows

$$A^a_\mu \to \zeta_\mu{}^a\,. \tag{3.1}$$

Such a correspondence is compatible with extended supersymmetry and can be used naïvely to promote $\mathcal{N} = 4$ super-Yang-Mills invariants into higher-derivative $\mathcal{N} = 8$ super-invariants. However it is not possible to capture all of these by such a map [22] since the full diffeomorphism invariance carries even more symmetry (For example Ricci cycling identities $R_{\mu[\nu\rho\sigma]} = 0$). The naïve application of the above substitution rule would for instance lead to a $\mathcal{N} = 8$ supergravity amplitude with an apparent factorisation of the operator D^2R^4. This would consequently make the $L = 2$ two-loop four-graviton amplitude diverge $D = 6$ dimensions. However this is contrary to explicit knowledge since this amplitude has been shown to be finite up to $D \leq 6$ dimensions [5].

The main reason for this glitch is that diffeomorphism invariance for $\mathcal{N}=8$ supergravity implies that one must sum over all permutations of external legs. Doing this makes the invariant $D^2R^4 \sim (s+t+u)\,R^4 = 0$ vanish by on-shell momentum conservation. Consequently the first non-vanishing contribution is of order D^4R^4. At two loop order the operator D^4R^4 satisfies the rule $\beta_2 = 2$ of eq. (2.6) providing the suitable structure for the two-loop amplitude kinematic factor. However this does neither give the correct contribution for the higher-loop amplitudes which have $\beta_L \geq 3$ for $L \geq 3$ [10, 13].

Thus we see that one has to be careful with such arguments since gravity theories have symmetries which are beyond what is provided via two copies of the gauge transformations of Yang-Mills theories.

An important consequence of the full crossing symmetry provided via the absence of the concept of colour in gravity theories is that infrared divergences in quantum gravity can be treated as in QED and are much milder than in colour ordered theories like QCD. One benefit of the structure of gravitational interactions is that there are no divergences for the emission of a soft graviton from a hard line contrary to massless QED [28]. This indicates that although gravitational interactions looks much more complicated than gauge theory ones important simplification occurs in on-shell amplitudes at tree-level. The fact that gravity amplitudes are unordered implies the no triangle property of $\mathcal{N}=8$ supergravity loop amplitudes [8, 9] and puts non-trivial constraints on the structure of higher-loop amplitudes [29].

4. Vacuum structure and E_7 invariance

The $\beta_L = L$ rule can be derived up to six loops from the zero mode sector of the pure spinor formalism [30] for four-graviton amplitudes. This shows that the rôle of extended supersymmetry in perturbative $\mathcal{N}=8$ supergravity are beyond the superspace transformation properties of the product of two $\mathcal{N}=4$ super-Yang-Mills theories. Further analysis show that the vacuum structure of $\mathcal{N}=4$ super-Yang-Mills and $\mathcal{N}=8$ supergravity theories are very different. Mathematically in four dimensions the vacuum of $\mathcal{N}=8$ supergravity can be described by the homogeneous space of $\mathcal{M} = E_{7(7)}/(SU(8)_R/\mathbb{Z}_2)$. While the local symmetry group $SU(8)_R$ transform as a 'square' of the group $SU(4)_R$ (corresponding to each $\mathcal{N}=4$ super-Yang-Mills theory) there is no concept of the symmetry of the global group $E_{7(7)}$ in $\mathcal{N}=4$ super-Yang-Mills. The global $E_{7(7)}$ symmetry does put severe enough constraints on counter terms $\mathcal{N}=8$ in supergravity to possibly protect the theory from diverging before nine loops and in conjunction with the full crossing symmetry this could be enough to imply finiteness of the theory in four dimensions. As a global symmetry rotating the different vacua of $\mathcal{N}=8$ supergravity the $E_{7(7)}$ symmetry relates the perturbative contributions to the non-perturbative black hole production at high-energy, which are required for a consistent definition of the theory [31].

5. Discussion

Although we are still in the search for a fundamental theory of quantum gravity we are in these years gaining a much better understanding of the necessary concepts for a formulation of such a theory. The rôle of dualities in supergravity theories is important for their quantisation and such investigations provide a framework for gathering further knowledge about quantum gravity, its fundamental degrees of freedom and its relation to gauge theory. A clear understanding of the question of ultraviolet finiteness and the validity of the $\beta_L = L$ rule [13] of $\mathcal{N} = 8$ supergravity would indeed be remarkable and provide huge implications for non-supersymmetric low-energy descriptions of quantum gravity theories.

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