In memoriam: Cécile DeWitt-Morette

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Remembering Cecile DeWitt-Morette

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I. The eventful life of Cécile (1922-2017)
II. My collaboration with Cécile

I. The eventful life of Cécile (1922-2017)

Cécile was born on December 21, 1922, in an apartment of the Ecole des Mines[1] facing the beautiful Luxembourg Garden in Paris. Her father was an important industrialist who died early. Her mother remarried with the principal collaborator of her husband. Cécile spent the first years of her life in Mondeville, in Normandy, followed by a year in Paris, where she was enrolled in second grade with her future friend and co-author Yvonne Choquet-Bruhat. After the year in Paris, her family moved to Caen, also in Normandy. She lived in Caen through most of World War II, and happened to be safe in Paris taking a physics exam on D-Day, but a bomb fell on her home in Caen and killed her grandmother, her mother, her beloved 16-year old sister, and the cook. We have to recall that Caen was heavily bombed in June 1944 by the Allied forces. Moreover, after the complete invasion of France, in the fall 1942, by German and Italian troops, France was divided into 8 different zones and an "Ausweis" (= passport) was required for interzone travel. Cécile had been given permission to travel between Caen and Paris to attend de Broglie's lectures. That explains how she escaped death.

After the war, she began working on a thesis under L. de Broglie. Soon after, she was hired by Frédéric Joliot(-Curie). Joliot, who, after surviving with some difficulty during the Nazi invasion of France, had become the head of the France nuclear program, as well as a fanatical communist militant, was too busy to prepare his lectures at Collège de France, and delegated the preparation to the rather inexperienced Cécile. Then she visited Great Britain and Ireland with the mission of interviewing important physicists about their work during the war. After a memorable encounter with Dirac, more talkative than customary, she visited the Institute for Advanced Study in Dublin. There she met E. Schrödinger, who had escaped from nazi Austria with his "two wives" and was welcomed by the Cardinal! There was also W. Heitler, who suggested to Cécile to work on meson physics, especially the distinction between π and μ mesons. That was the subject of her thesis[2], defended in front of de Broglie, after her return in Paris.

While in Copenhagen, Cécile received a telegram from Robert Oppenheimer, which read, "On the recommendation of Bohr and Heitler, I am glad to offer you membership in the school of mathematics Institute for Advanced Study (IAS) for the academic year 1948-1949 with a stipend of $3500.00." Although Cécile had no idea where Princeton was, she accepted the invitation and there began her lifelong friendship with Freeman Dyson, and a more complicated relationship with Richard Feynman[3]. It was at the IAS, the following year, that she met her future husband, Bryce Seligman DeWitt, a "Schwinger boy" and veteran of the US Navy, in which he had been trained to be a naval aviator during the war. Cécile initially rejected Bryce's marriage proposal and accepted only after she came up with the idea, literally

[1]If I remember correctly, her grandfather was the general secretary of the school.
[2]With title: "Sur la production des mésons dans les chocs entre nucléons".
[3]His relationship with women was always complicated!

overnight, of creating the Les Houches summer school as her penance for marrying a foreigner and as a way to contribute something back to France.

At the time, in the fifties, experimental physics (and chemistry) was still strong in France, mainly as part of the Curie heritage. Nuclear physics, magnetism (remember the Weiss' domains[4]), solid state physics (created by Pierre Aigrain and Alfred Kastler and developing later into ultralow temperatures by Claude Cohen-Tannoudji and his school). On the other hand, theoretical physics was in a poor state. The school of de Broglie, filled with incompetent and sometimes dishonest people, went into a deadlock by refusing the "Copenhagen interpretation" and developing tools like "the double solution" leading nowhere. Louis de Broglie considered that quantum mechanics was like a luxurious jewel, to be reserved for the happy few. As a result, we had to wait until the late sixties for the creation of regular graduate (and then undergraduate) courses about quantum mechanics. In the fifties, neither Landau and Lifschitz' textbook nor the Feynman lectures were available, and I learned quantum mechanics from the giant books of Hermann Weyl and P.A.M. Dirac.

Gravitation and General Relativity were not in a better situation. The founding fathers in France, namely Georges Darmois, Marie-Antoinette Tonnelat, and André Lichnerowicz were skilled mathematicians, who developed it like a purely mathematical theory. Yvonne Choquet-Bruhat came later with more interest in physics. I remember discussing around 1970 with Lichnerowicz the first experiments by Weber. He was not at all interested despite his own mathematical contributions to the existence of gravitational waves. It was the team of theoretical physicists at Meudon (Brandon Carter, Thibault Damour) who transformed, within the French community, General Relativity into a physical discipline[5].

To fill this vacuum, Cécile came up with a very original and bold idea, to create a summer school in physics, where, in the inspiring place of Les Houches in the Alps, students and professors would share life and science for two months. This was started in 1951 and brought lecturers like Pauli or Feynman. The place has been a landmark in theoretical physics for 70 years.

For me, it was a "missed opportunity", which influenced my own career. The first sessions were attended by some of my classmates, like Pierre-Gilles de Gennes and Bernard Malgrange. I was then 22 and full of youthful dreams, ranging from theology (I had just given up the idea of becoming a minister in the French Calvinist Church), to unorthodox Marxist philosophy (following my professor of philosophy Louis Althusser) and to radioastronomy (then a new science). But in between, I had been seduced by Bourbaki, and put all of my energy in their ambitious program of reconstructing mathematics from scratch.

After Princeton and the marriage of Cécile and Bryce, came the period of Chapel Hill. For 16 years she was unable to get a position there, because of a rule forbidding a married couple to be hired in the same department. She was the mother of four daughters, as efficient as a mother as in any of her enterprises. One of her daughters has a mental illness. She was a big concern for Cécile, but when you are Cécile, the answer is to become actively engaged in mental health endeavors, establishing a local mental health support organization, and organizing fundraising events such as one honoring a prominent caregiver, Alicia Nash, who cared for her husband John Nash and their son through many years of their respective illnesses.

Then, in 1972, came the fortunate move from Chapel Hill to Austin. Austin is a very

[4]Henri Cartan was the son in law of Weiss!

[5]It took fifty more years to realize the dream of observing these gravitational waves!

pleasant campus and downtown[6]. On both gown and town you have a wealth of theatrical and musical events. Besides the "President Johnson Library" and the football field, you have a big auditorium seating 3000 persons, where you can watch the Covent Garden, three nights in a row, for $20 a night.

Despite Texas' climate, the town is green, with many parks, an artificial lake and a "natural" pool in a canyon, where you swim in the midst of turtles (and perhaps snakes!). To sum up, Austin, both campus and town are liberal within an harshly conservative Texan environment. A standard saying is: "If Texas secedes[7] some day from the United States, the day before Austin will secede from Texas".

Cécile was at first appointed to the Department of Astronomy. The dominant figure there was Gérard de Vaucouleurs[8], a French émigré, who together with his wife maintained and developed a catalog of galaxies. Now that we estimate the number of galaxies to be of the order of 10^{11} (each galaxy with 10^{10} stars in the mean), I don't know the purpose and meaning of such a catalogue today. Many observations were done at the MacDonald Observatory, and people commuted between Austin and MacDonald (a roundtrip drive of 1500 km in the Texas desert). Nowadays, only people in charge of maintaining the instruments visit MacDonald, and astronomers watch the sky through their computer screen.

After some years -in 1983- she joined the Physics Department. This was a strong department, chaired at the time by Tom Griffy. Yuval Ne'eman and Ilya Prigogine were part-time faculty members. Prigogine had received the Nobel Prize in 1977. Texas physics Professor Steven Weinberg, also a Nobel laureate, is a superb scientist, with a list of beautiful books (including one in cosmology), but a strong character who claims to know everything[9]. Despite political differences[10], I became a friend of Ne'eman and we shared students and seminars. On the other hand, Prigogine was disappointing. When he announced a seminar talk, it was eventually delegated to one of his assistants. Nonequilibrium physics is a fascinating subject, but he used the wrong tools for that[11].

A feature of the Physics Department at that time was its strong research centers. One was the Center for Relativity, which had been created by Alfred Schild. Bryce DeWitt was the Director in 1983. Cécile was a member of that Center. There was also the Center for Theoretical Physics, created by John Archibald Wheeler, after his retirement from Princeton University. And Steven Weinberg headed a third center, the Theory Center. At this point, Wheeler was interested in Quantum Gravity, as was Bryce DeWitt, who had been interested in Quantum Gravity from the beginning. There was another character in this center, whom we called the "captain" because he was later on associated with joint experimental programs

[6]Not everyone realizes that Austin is a dual city, the Mexican one on the eastside, and the "WASP" one on the westside, separated by the physical barrier of Interstate 35. Now that the airport has moved to the East side, visitors have more difficulties ignoring this geographical feature. Also, in the "Cinco de Mayo" (5 May, Mexican national festival), the town belongs to the Mexicans. Notice also that, in an academic building after 8 p.m., if you need a key, better ask about it in Spanish!

[7]I doubt that after the civil war, Texas would be permitted to secede, but there is a bylaw allowing Texas to split into five pieces, each one sending two senators to Washington, D.C. A similar bylaw applies to California.

[8]The Vaucouleurs had a very gifted technician, who decided once to run with his daughter a (very successful) business of French bakeries. Before leaving, he had to swear to visit Austin for a week every year: he was the only one able to unlock some secret tools!

[9]It took me time and flexibility to be respected by him. I was helped in this endeavor by a visit of Ludwig Faddeev, who treated me like an old friend!

[10]Ne'eman had many political and military responsibilities in Israel, and is the father of Israel's nuclear weapons program. In my opinion he would have deserved to be a Nobel laureate, but perhaps the Nobel committee was afraid of political controversies.

[11]I once advised a graduate student in his group to go elsewhere after being granted his Ph.D.

of the Navy and NASA (first versions of the GPS system!).

Both Cécile and Bryce were adventurous characters. Before telling their adventure in Mauritania to watch a solar eclipse, I have to insert a comment on the experimental basis of Einstein's gravitation theory. Three tests of General Relativity have been historically important:

a) *The residual motion of Mercury's perihelion* ($43''$ of arc per century!). Despite many efforts, and many proposals (a new planet, Vulcan, between the Sun and Mercury, an oblate Sun, with quadrupolar effect, modification of Newton's law) no astronomical explanation was successful, while Einstein's calculations in 1915 fitted the data very well.

b) *The redshift of spectral lines.* Stellar data were quite inconclusive, and only through the use of the Mossbauer effect on earth the experiment of Pound and Rebka in 1959 confirmed this redshift, now a central element in astrophysics!

c) *The bending of light rays around the Sun.* Here there is a crucial factor of 2. According to a calculation going back to Laplace, and assuming that a light ray corresponds to the motion of a particle (photon?), the gravitational influence of the Sun results in a Keplerian hyperbola, independent of the mass of the photon by Galileo's universality of free fall. The deviation, that is the angle between the two asymptotic directions, is 2α radians, in the first order in $\alpha = \frac{GM_\odot}{bc^2}$ (G gravitation constant, c speed of light *in vacuo*, b impact parameter, $M_\odot$ mass of the Sun). Numerically $\alpha \sim 2 \times 10^{-6}$ for a ray grazing the Sun.

In 1907, Einstein tried for the first time to account for the influence of gravitation on light. He assumed that at a place $\mathbf{x}$ where the gravitation potential is Φ, the speed of light is $\sqrt{c^2 + 2\Phi}$ (a consequence of the conservation of energy for a particle!). To include relativity effects, we have to modify the Minkowski metric $\eta = c^2\, dt^2 - d\mathbf{x}^2$ by replacing c^2 by $c^2 + 2\Phi$. If we do not change the spatial metric $d\mathbf{x}^2$, we get in first approximation in α the same result, 2α radians, as the one given by the above-quoted Newtonian reasoning. In polar coordinates, we have $d\mathbf{x}^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta \cdot d\varphi^2)$. But taking in full account the field equations for gravitation (discovered by Einstein in November 1915!), we have to replace dr^2 by $dr^2/\left(1 + 2\frac{\Phi}{c^2}\right)$ with the gravitational potential $\Phi = -GM_\odot/r$ (Schwarzschild). To calculate the deviation to first order in α, it is enough to use an approximate solution discovered by Einstein in 1915. If we take this into account, the deviation becomes $2 \times 2\alpha$ radians (equal to $1.75''$ of arc). Here is the factor of 2. Eddington in 1919 claimed to have observed this.

Let us go back to Bryce and Cécile! They decided to improve on previous measurements of light deflection by taking advantage of the 1973 solar eclipse, visible in Mauritania. Their equipment was transported by the U.S. National Center for Atmospheric Research, which arranged a large ($\sim$ 50 person) eclipse expedition to a desert oasis, Chinguetti, Mauritania, including six scientists carrying out the deflection of light measurement (among them Cécile and Bryce.)[12] Their observations confirmed Einstein's prediction to 10%.

Summarizing, there was little solid observational basis for General Relativity before 1960. Things have changed: within the solar system the gravitational field is of order α at most (weak field) but the post Newtonian approximation program, with data fitting numerous space observations, gives an excellent agreement. For strong fields, binary pulsars and more recently, gravitational waves, give unambiguous, accurate confirmations.

Before describing our collaboration, let me give a few examples of the managerial ability

[12]Let me add that Bryce was a well-seasoned pilot from his war experience. In the beginning of space exploration, he volunteered and went through the physical tests, only to be turned down because he was over forty.

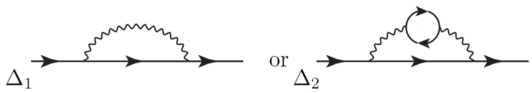
of Cécile.

Once in Les Houches, during one of my attendances there, there was no one to serve breakfast on a Sunday morning: the two local aids in the kitchen had been out dancing for the whole night. Ten minutes later, everything was in order with Cécile herself preparing coffee and toast!

Much later, there were discussions about recruiting her for the board of trustees of IHES, and some people objected about her age, being afraid that she would sleep during the meetings. I told everyone that no one can sleep in a meeting where she is present! Indeed, she usually came a few days in advance, meeting everyone on the staff to know everything by herself. One of her last scientific works was a publication of Bryce's papers[13].

II. My collaboration with Cécile

It lasted more than 30 years and ended with the publication of a book[14]. It started in two distinct ways. When Cécile visited Princeton in the 1950's the latest development in physics was *Quantum Electrodynamics*. What was needed was a mathematical model to account for the newly observed Lamb shift, at a deeper level account for the radiative corrections to atomic spectra. In classical terms, the reaction on an electron of mass m of the electromagnetic field emitted by this electron is infinite (for a pointlike electron). In quantum theory, the effect is still infinite, but with a logarithmic (and not linear) divergence. Bethe proposed a *cutoff*: the frequencies higher than a bound $\Lambda = \frac{mc^2}{h}$ should be ignored. The challenge was to make it compatible with (special) relativity. Using a very ad hoc proposal ("path integrals" or "sum over histories"), R. Feynman gave a recipe using his well-known diagrams. For instance, for one incoming and one outgoing electron, intermediate stages can be pictured by diagrams like



(solid lines for electrons, wavy lines for photons). If the incoming (and outgoing) 4-momentum is $\mathbf{p}$, then each diagram Δ contributes an amplitude $A_\Delta(\mathbf{p})$ and one estimates probabilities by $\left|\sum_\Delta A_\Delta(\mathbf{p})\right|^2$. Feynman gave explicit formulas to calculate $A_\Delta(\mathbf{p})$ by means of integrals. The complexity of these integrals increases with the number of *loops* in the diagrams (1 for Δ_1, 2 for Δ_2, etc...). The result can be expanded into a power series in the coupling constant $\alpha_e = \frac{e^2}{4\pi\varepsilon_0\hbar c}$. Each such integral diverges logarithmically, but putting a Bethe-type cutoff Λ makes them convergent, and the recipes of renormalization tell us how to go to the limit $\Lambda \to \infty$.

That was an immediate success, most notably by the calculation of the Lamb shift and anomalous magnetic moment of the electron. The intuitive character of the diagrams made them very popular, but Feynman's justification by path integrals seemed ad hoc, and was ignored by most people. Soon after, F. Dyson rediscovered under the name of "time-ordered exponential series" a familiar tool in ordinary differential equations (Lappo-Danilevsky), and gave an *ab initio* derivation of Feynman's formulas.

Some years later, Feynman gave a basic course about path integrals, which eventually was written up by his student Hibbs and published under their two names. This was not

[13] *The Pursuit of Quantum Gravity. Memoirs of Bryce DeWitt*, Springer, 2000.

[14] P. Cartier and C. DeWitt-Morette, *Functional integration (Action and symmetries)*, Cambridge Univ. Press, 2006.

a success, and the book remained very unconvincing. Why? I don't know. There were numerous attempts by both physicists and mathematicians, in the West as well as in Soviet Union, to formulate correctly the path integrals. But before describing the proposal made by Cécile and myself in our book, let me describe the other development, running in parallel: *history of measure theory and Lebesgue integral.*[15]

Lebesgue integral was invented around 1900, and underwent many developments until 1940. A notable event was the proof by Markoff in 1938 of a vast generalization of previous results of J. Hadamard and F. Riesz. This result establishes a bijection, between regular Borel measures[16] μ on a compact space X, and the linear forms $\overline{\mu}$ on the Banach space $C^0(X;\mathbb{R})$ of continuous real-valued functions on X, which satisfy $\overline{\mu}(f) \geq 0$ for $f \geq 0$. The correspondence is given by $\overline{\mu}(f) = \int_X f \cdot d\mu$. As a bonus, the Lebesgue space $L^p(X,\mu)$ is the completion of the space $C^0(X;\mathbb{R})$ under the norm $N_p(f) = \left(\int_X |f|^p d\mu\right)^{1/p}$. This result is readily extended to a locally compact space, and A. Weil saw this duality approach as the right foundation for a general integration theory. His views were supported within Bourbaki by L. Schwartz, who liked the idea of duality and used it in the creation of the theory of distributions. R. Godement put all of his energy to pursue the project which ended by the publication of Bourbaki in several volumes around 1950.

The method worked well for *locally compact* spaces. But Bourbaki was obsessed by the idea of removing any countability assumption from topology and analysis – at the cost of rather akward definitions, like the one of measurable sets or functions (also the various definitions of the exterior measure of a set!). The limitation to locally compact spaces didn't look serious for geometrical and even number-theoretical applications.

As long as probability theory focused on finite (or even countable) families of random variables, one needed only integration over finite-dimensional manifolds (hence locally compact spaces) but the rapid development of random processus required new tools, and Bourbaki decided to complete his treatise on integration. This was done in the 1960's. Officially, every text in the Bourbaki treatise is anonymous, and it is really a team work. Today, I can say that the impetus came from L. Schwartz, P.A. Meyer and myself. Schwartz had as father-in-law, Paul Lévy, a famous probabilist, P.A. Meyer was then a star among French (and world) probabilists, and myself was motivated by my interest in probability and mathematical physics. After many strenuous efforts, it was generally accepted that the right category of spaces were the *Polish spaces* (separable, complete, metric) or some slight generalizations (Souslin or Lusin spaces). Borel measures were automatically regular and the Soviet School (Gelfand, Prokhorov, Minlos, ...) had established the existence of infinite products of measures, or more generally projective limits. There a compactness criterion, given by Prokhorov, gives the final answer.

The most innovative section in the new chapter of Bourbaki deals with *promeasures*, a linear version of the general method sometimes called cylindrical measures[17]. Following S. Bochner (1955) and generalizing a method invented by P. Lévy in the 1920's one introduces the *Fourier transform* of a promeasure in the linear space X, a function on the dual space

[15] A readable account is given by Bourbaki (see *"Eléments d'histoire des mathématiques"*, Hermann, Paris, 1960, 2$^{\text{nd}}$ édition, Springer Verlag, 2007).

[16] A Borel measure on a compact space X is a nonnegative σ-additive function μ on the σ-algebra of Borel subsets of X. Regularity means that for any measurable subset E of X and any $\varepsilon > 0$, there exist two subsets, K compact, U open such that $K \subset E \subset U$ and $\mu(U) - \mu(K) \leq \varepsilon$. Regularity is automatic if there is a countable basis of open sets in X.

[17] This corresponds to the habit in probability theory, of describing the law of a process $(X(t))_{t\in T}$ by giving the *marginals*, that is the law of random vectors $(X(t_1)\dots X(t_n)) \in \mathbb{R}^n$ extracted from the random process.

X' of X. By Minlos theorem, if the space X is *nuclear* in the sense of Grothendieck's thesis, the continuity of the Fourier transform on X' is what is needed to have a true measure on X, instead of the approximating promeasure.

Here the two trends came together. It started in an exchange between J. Dieudonné and Cécile in 1971. She was mentioning to him that in physics one needs oscillating integrals like $\int_{-\infty}^{+\infty} e^{ix^2} dx$ instead of convergent integrals like $\int_{-\infty}^{+\infty} e^{-x^2} dx$. Dieudonné replied that Fourier transforms exist for distributions and take care of some of the oscillating integrals. His suggestion was to replace *pro-measures* useful in probability theory by *pro-distributions* and to ask Cartier about that. A week later, I met Cécile at a party at IHES, and so started our long-term collaboration and friendship. So, I could repair the "missed opportunity". At last my investment in writing the Bourbaki treatise could bring fruits for our collaboration.

Our scientific marriage lasted more than 30 years. Cécile had already published two books[18] in collaboration with Yvonne Choquet-Bruhat and the first idea was a book by her and her husband, together with mathematical appendices by myself. This could not be done for scientific and personal reasons, and soon we agreed on writing together a book[19], while Bryce DeWitt published his own version[20]. It required me to visit Austin every year, visits I enjoyed very much. I was the guest of both the Physics and Mathematics departments. On both sides, I had good connections, most notably John Tate on the Math. department. I shared graduate students with Cécile. I particularly remember a young Chinese woman by the name of Xiao-Rong Wu, who needed less than a year to transform from a shy Chinese woman into a standard American student, marrying an American. Also, a German by the name of Saemann (whom I met recently in Edinburg) and Markus Berg a Swede who came back to his country because he was not a member of the "string fashion" in the USA; he offered a wonderful wedding party in Austin. Cécile and myself organized many scientific events in Les Houches, Cargese, Institut Henri Poincaré, Strasbourg, etc. ... My wife and I visited Texas, Louisiana, New-Mexico in memorable travels and as mentioned already, I enjoyed life in Austin: bicycle, swimming, concerts, theater plays, the bats under the bridge on the lake, ... Too much to give here details. For all that, including the rewarding enterprise of writing a book:

MERCI CÉCILE

[18]Y. Choquet-Bruhat and C. DeWitt-Morette, *Analysis, Manifolds and Physics*, Part I: *Basics* and part II: *92 applications*, North Holland, 1982 and 1989.

[19]For a technical description of this book, see the companion note "A tutorial in Feynman path integrals".

[20]B. DeWitt, *The global approach to Quantum Field Theory*, Oxford Univ. Press, 2004.