The physics of quantum gravity[☆]
La physique de la gravitation quantique

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Abstract

Quantum gravity is still very mysterious and far from being well understood. In this text we review the motivations for the quantification of gravity, and some expected physical consequences. We discuss the remarkable relations between scattering processes in quantum gravity and in Yang-Mills theory, and the role of string theory as an unifying theory.

Résumé

Comprendre la physique de la gravitation quantique est un enjeu majeur de la physique moderne. Dans ce texte nous exposons des raisons en faveur de la quantification de l'interaction gravitationnelle, et nous décrivons quelques conséquences physiques attendues. Nous discutons les relations remarquables entre amplitudes de diffusion en gravité quantique et théorie de Yang-Mills, ainsi que le rôle de la théorie des cordes comme théorie unificatrice.

Keywords: scattering amplitudes, string theory, quantum gravity

1. The standard models of particle physics and cosmology

The recent confirmation of the Brout-Englert-Higgs mechanism [1, 2] by the ATLAS and CMS experiments [3, 4] at CERN is a great success for

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the standard model of particle physics [5]. The increased precision in the measurement of the structure and dynamics of the observable Universe by the Planck experiment [6] is putting the standard model for cosmology on a solid ground [7].

The standard model of particle physics is a beautiful theory that accounts for all the phenomena observed in accelerator physics. It is based on the formalism of quantum field theory for local continuous symmetries: the space-time invariance under orientation and the boost velocity, and the internal symmetry group describing the strong force of Quantum Chromodynamics (QCD), the weak interactions, and the electromagnetic force. Experiments confirm that the interactions between elementary particles are carried by vector particles: the gluons for the strong force, the photon for the electromagnetism, and the massive (thanks to the Brout-Englert-Higgs mechanism) bosons W^+, W^- and Z^0 for the weak interactions.

The standard model is mathematically consistent because renormalisable, but it does not explain all observed phenomena, like the origin of the neutrino masses or the asymmetry between matter and antimatter [8]. Observations indicate that only 4% of the mass of the observable Universe are seen, and that dark energy and dark matter are required.

These results provide strong confirmations of the current models of particle physics and cosmology, but they strengthen our opinion that more fundamental models are needed.

2. Beyond the standard models

A popular extension of the standard model of particle physics is the introduction of a new symmetry of space-time the so-called supersymmetry [9]. The matter constituents, the electron, the quarks, &c, are of half-integer spins. The particles responsible for the interactions are of integer spin. Supersymmetry is a new symmetry associating to any elementary particles of integer spin (the bosons) a partner of half-integer spin (the fermions) and vise-versa.

One can see supersymmetry as the introduction of a new set of anticommuting fermionic coordinates $\theta_\alpha\theta_\beta = -\theta_\beta\theta_\alpha$ with $\alpha, \beta = 1, \ldots, 4$ in addition to the usual four space-time coordinates $x^\mu = (t, x, y, z)$ where t is the time and (x, y, z) are the spatial coordinates. A translation along the fermionic coordinates $\theta_\alpha \to \theta_\alpha + \epsilon_\alpha$ induces a space-time translation [9]. These new coordinates are quantum dimensions with no classical analog.

In a supersymmetric theory, the photon has a partner the gaugino that participates in the interactions between charged particles. Similarly, the weak and strong forces are modified by the participation of the supersymmetric partners of their force carriers. Therefore the energy dependence of the quantum interactions is modified. Particular supersymmetric extensions of the standard model lead to a unification of the forces at an energy of the order $E_{\rm GUT} \simeq 10^{16}$ GeV [10]. Although this scale seems unreachable by direct accelerator physics experiments, it is remarkably close to the natural characteristic energy of quantum gravity effects which is determined by the Planck energy $E_{\rm Planck} = \sqrt{\hbar c^5/G_N} \simeq 1.22\,10^{19}$ GeV.

Despite the standard model of particle physics does not include the gravitational interactions, the proximity of these two scales points to the importance of quantum gravity physics in the early times of our observable Universe after the Big Bang, or in relation to the dark energy and dark matter puzzles.

3. The road to quantum gravity

Einstein had realized that, for Rutherford's classical atomic planetary model, an electron orbiting in an atom would lose energy by gravitational radiation, and would then fall on the nucleus. The mechanism is similar to the orbital shrinkage of the binary pulsar PSR B1913+16 by the emission of gravitational radiations [11], which constitutes one strong confirmation of Einstein's gravity theory. This is the same phenomena as the loss of energy by electromagnetic radiation implying a collapse of the electron on the nucleus in about 10^{-10} s. It is the quantification of the electromagnetic forces that assures the stability of the atom. Einstein calculated that the loss of energy by gravitational radiations would lead to the disappearance of atoms after about 10^{30} years. Although this is much longer than the 10^{10} years of the age of our observable Universe, this convinced Einstein of the need of a quantum formulation of gravity [12], and the necessary unification of all fundamental interactions.

Although Einstein's theory of general relativity for gravitation is extremely well tested within the solar system, or by the binary systems [13, 14], tests on larger and much smaller scales are not so stringent. This leaves a lot of room for considering various new gravitational effects.

Sakharov [15] has pointed out that quantum fluctuations from matter fields induce Einstein's gravity, and in the AdS/CFT correspondence the degrees of freedom of quantum gravity in the bulk of space-time induce the ones on the boundary [16].

It has been argued in [17, 18] for the inconsistencies of the interactions between a classical gravitational field and quantum-mechanical matter. Dyson has reexamined this in a text [19] where he is asking if a single graviton, the quantum of the space-time waves, would be detectable in the same way as we detect the photons, the quantum of light waves.

Therefore, it looks a reasonable assumption that gravity should be quantized as the other fundamental forces.

One approach to quantum gravity is to follow a treatment along the line of the quantum field theory formalism used in particle physics. Following Feynman [20, 21] and DeWitt [22], one quantizes the graviton field with quanta described by a massless spin 2 particle $h_{\mu\nu}$ identified as the fluctuations of the gravitational field $g_{\mu\nu} = g_{\mu\nu}^{\text{classical}} + \sqrt{32\pi G_N/c^2}\, h_{\mu\nu}$. Where G_N is Newton's constant for the strength of the gravitational force.

This approach of quantum gravity immediately faces the important problem of the bad high-energy behaviour of the theory. The scattering cross-section of two gravitons of opposite polarization diverges at high-energy $d\sigma/d\Omega \propto (G_N E)^2$ and the non-renormalisability of perturbative quantum gravity was shown by 't Hooft and Veltman [23].

Because the fluctuations of the gravitational field represent the fluctuations of the space-time metric, any tiny modification of the physical laws at the Planck scale $\ell_P = \sqrt{\hbar G_N/c^3} \simeq 10^{-35}$ m would lead to a change in the propagation of light with a blurring of the spectral lines. Recent measurements of Gamma ray bursts [24, 25, 26] put strong constraints on possible violations of Lorentz invariance at the Planck scale.

The formulation of gravity in models with many new quantum fermionic coordinates θ^i_α with $i = 1, \dots, \mathcal{N}$, leads to the so-called supergravity theories [27]. The more fermionic coordinates the more new fields are introduced that participate in the interactions. If we are willing to consider theories with many vector interactions, like the standard model, there is no consistent model with several gravitons [28]. This is satisfying since the graviton represents the quanta of space-time waves. Consequently, one cannot add more than eight $\mathcal{N} = 8$ families of fermionic coordinates [29]. The maximal supergravity theory constructed in [30, 31] has an improved high-energy behaviour [32, 33]. This raised the question if this theory could provide a consistent theory of quantum gravity, without any need of extra high-energy degrees of freedom. This question will be discussed in section 6.

String theory takes another route for addressing the problems of the unification of fundamental interactions with gravity. It posits the propagation of fluctuating strings of tension $T_s = 1/(4\pi\ell_s^2)$, the dynamics of which is described by a two dimensional gravity action [34]

$$S_{\rm string} = -\frac{1}{4\pi\ell_s^2}\int_\Sigma d^2\sigma\,\sqrt{\det h}\,\left(h^{ab}g_{\mu\nu}(X)\partial_a X^\mu(\sigma)\partial_b X^\nu(\sigma) - 2\ell_s^2\Phi\,\mathcal{R}_{(2)}\right)\,. \tag{1}$$

The matter fields $X^\mu(\sigma)$ are the coordinates for the embedding in a space-time of the surface Σ swept by the string. The dynamics of gravity in two dimensions is rather trivial and the Einstein-Hilbert term $\mathcal{R}_{(2)}$ only contributes through the Euler characteristic $\chi(\Sigma)$, which is a topological invariant of the shape of the surface Σ regardless of the way it is bent. The sum over all the fluctuating geometries is organized as a sum over the topologies weighted by the factor $\exp(-\langle\Phi\rangle\chi(\Sigma))$ where the vacuum expectation value of Φ measures the strength of the interactions between the strings. When propagating in a space-time geometry specified by the metric $g_{\mu\nu}(X)$, all fluctuations of size smaller than the typical string length ℓ_s are smoothened [35].

Scherk and Schwarz have shown [36] that in the limit where the string length goes to zero, $\ell_s \to 0$, one recovers Einstein's theory of gravity. For the maximally supersymmetric string theory one recovers the maximal supergravity theory [34]. It is amusing that string theory uses a two dimensional quantum gravity theory to address the problems of quantum gravity in space-time.

4. Quantum gravity effective field theory

Without knowing the nature of the fundamental microscopic degrees of freedom of gravity one can nevertheless treat quantum gravity as an effective theory [37]. An effective field theory is a technique to separate the high-energy scales from the low-energy scales, and to treat the resulting theory as a standard (non-renormalisable) quantum field theory. The scattering amplitudes describing the interactions between elementary particles are constrained by the usual criteria of quantum field theory: unitarity, locality and gauge invariance.

Quantum gravity processes give rise to local contributions associated with small scale high-energy ultraviolet behaviours, and infrared effects modifying the interactions at large distances. Infrared physics does not depend on the fine details of the high-energy physics, and the question of the non renormalisability of the theory is not anymore too important.

The evaluation of the gravitational interaction between two static masses m_1 and m_2 at a distance $|\vec{r}|$ leads to corrections to Newton's potential [37, 38]

$$V(r) = -\frac{G_N m_1 m_2}{|\vec{r}|}\left(1 + 3\frac{G_N(m_1+m_2)}{|\vec{r}|c^2} + \frac{41}{10\pi}\frac{G_N \hbar}{\vec{r}^2 c^3}\right) + K\,\frac{\hbar G_N^2}{c^3} m_1 m_2 \delta^3(\vec{r})\,. \tag{2}$$

The local contribution $\delta^3(\vec{r})$ is due to the high-energy behaviour and the value of the coefficient K depends on the high-energy degrees of freedom. The $1/\vec{r}^2$ correction is the first *classical* post-Newtonian contribution from the general relativity. This contribution is independent of the high-energy degrees of freedom [37, 39]. The $1/|\vec{r}|^3$ contribution is of quantum nature but only depends on the low-energy modes, and must be reproduced by *any* theory of quantum gravity.

5. Perturbative gravity as the square of Yang-Mills theory

One would like to understand the energy dependence of the emission of gravitons, and how gravity affects particle physics processes. In particular quantum gravity signals are being searched at the Large Hadron Collider (LHC) at CERN [40, 41].

When smashing two incoming particles at high energy, a multitude of new outgoing particles are created. This physical process is analyzed by computing scattering amplitudes in the perturbative regime where the strength of the interactions is small. One then deduces a cross-section compared to the measured data. The methods of computing scattering amplitudes based on technics introduced by Feynman can turn to be very involved [42]. Even for the elementary QCD processes of two gluons leading to n gluons at tree-level order (the leading order in perturbation), the number of contributions to evaluate grows without control [43]

n gluons	2	3	4	5	6	7	8
# diagrams	4	25	220	2485	34300	55405	10525900

The situation with perturbative quantum gravity computations is even worse: terrible technical difficulties make the computation of simple processes hopeless. Motivated by the search for new physics at LHC, new powerful methods to evaluate analytically and numerically scattering amplitudes have been designed [44]. This was needed in order to confront the experimental data with the current models and possibly discover new phenomena. These new methods are based on the fundamental properties of quantum field theory: Unitarity, Lorentz invariance, and gauge symmetry invariance.

Since string theory reduces at low-energy to standard Yang-Mills theory and Einstein's gravity [34], one can contemplate using string based method for computing quantum gravity processes [45, 46].

For instance, the sum of the field theory tree-level n-particle amplitudes in Yang-Mills theory are obtained from a single string theory integral in the limit, $\ell_s \to 0$, where all the massive string excitations are decoupled

$$A_n^{\mathrm{Yang-Mills}}(g_1, \ldots, g_n) = \lim_{\ell_s \to 0} \int f(x_1, \ldots, x_n) \prod_{1 \leq i < j \leq n-1} (x_i - x_j)^{\ell_s^2 k_i \cdot k_j} \prod_{i=2}^{n-2} dx_i \,. \tag{3}$$

The properties of the string theory integral imply linear relations on the vector $\mathrm{A}^{\mathrm{Yang-Mills}}$ of tree-level Yang-Mills theory amplitudes [47]

$$\mathcal{S} \cdot \mathrm{A}^{\mathrm{Yang-Mills}} = 0 \,. \tag{4}$$

Where $\mathcal{S}$ is a singular matrix, so-called the momentum kernel, that depends only on the kinematic invariants of the considered process [47, 48]. The momentum kernel is the same for all types of interactions: QCD, QED, or supersymmetric Yang-Mills theory. The fundamental relation (4) can be used to check any numerical evaluation or any construction of the amplitudes by other methods. The momentum kernel formalism provides all possible relations between the tree-level quantum field theory amplitudes in Yang-Mills theory.

String theory provides as well a representation for the gravitational amplitudes, as the $\ell_s \to 0$ limit of a two dimensional integral

$$M_n^{\mathrm{gravity}} = \lim_{\ell_s \to 0} \int_\Sigma f(z_1, \ldots, z_n) \prod_{1 \leq i < j \leq n-1} |z_i - z_j|^{2\ell_s^2 k_i \cdot k_j} \prod_{i=2}^{n-2} d^2 z_i \,. \tag{5}$$

This representation leads to an expression of the tree-level quantum gravity processes as bilinear of Yang-Mills theory amplitudes [48, 47, 49]

$$M_n^{\mathrm{gravity}} = (\mathrm{A}^{\mathrm{Yang-Mills}})^T \cdot \mathcal{S} \cdot \mathrm{A}^{\mathrm{Yang-Mills}} \,. \tag{6}$$

This relation indicates that quantum perturbative processes in gravity and supergravity can be expressed in terms of sum of squares of processes in QCD and supersymmetric Yang-Mills theory [50, 51].

One remarkable consequence of (6) is that the gravitational Compton scattering — the scattering of a graviton g by a spinless matter target $Xg \to Xg$ — can be expressed as the product of the QED Compton scattering of a photon γ by the same spinless matter target $X\gamma \to X\gamma$ [52, 39]

$$M^{\text{gravity}}(Xg \to Xg) = \frac{G_N}{c^2 e^2} \frac{(p_1 \cdot k_1)\,(p_1 \cdot k_2)}{k_1 \cdot k_2} \left(A^{\text{QED}}(X\gamma \to X\gamma)\right)^2 , \qquad (7)$$

where p_1 and p_2 are the momenta of the matter field X and k_1 and k_2 the momenta of the gravitons. This relation implies that the computation of the quantum gravity correction to Newton's potential in (2) reduces to the evaluation of contributions obtained by squaring the quantum corrections to Coulomb potential in quantum electrodynamics [39].

Such a simple relation may look surprising because the gravitational force is a non-linear self-interacting theory, whereas there is no classical self-interaction between light. Actually Weinberg explained [53] that the infrared properties of gravity are similar to those of electrodynamics. Consequently in the first order of the perturbative expansion, the quantum radiation of gravitons and photons share many similar properties [54]. To summarize most of the difficulties of quantum gravity arise at high-energy.

6. High-energy and black hole production

In the previous section we have described how the formalism of string theory leads to fundamental relations between quantum field theory scattering amplitudes in the small coupling regime. In particular, we have described how this provides the rather surprising relations (4) and (7) expressing gravity scattering amplitudes as the square of Yang-Mills theory amplitudes.

Thanks to these striking relations, and the improved high-energy behaviour of the maximal supergravity theory, one could wonder if new high-energy degrees of freedom are really needed to get a consistent theory of quantum gravity [55].

The maximal supergravity theory has several copies of the electric and magnetic charges (q_e^i, q_m^i) with $i = 1, \ldots, 56$. At high-energy electrically and magnetically charged microscopic black holes are pair produced violating charge conservation. Some of these microscopic black holes are classical singular solutions without a horizon to hide the gravitational singularity.

In a quantum theory each set of electric and magnetic charges satisfy the Dirac quantization condition and $q_e^i q_m^i/(2\pi\hbar c^2)$ is an integer. The set of quantized charges defines a lattice which has a natural definition in string theory [56]. Removing any of the singular classical solutions leads to an inconsistent lattice of charges. The quantum fluctuations in string theory cloak the singularity with a horizon and provide consistent black hole solutions of Planck mass. The singular black hole solutions of maximal supergravity are the symptom that the theory is a singular truncation of string theory [57].

One needs to take into account the contributions of charged microscopic black holes arising at very high-energy. These effects are not captured by the methods described in section 5 which are valid in the regime of small gravitational interactions. Outside this regime one needs to use fundamental properties of string theory. Direct computations are difficult but they are greatly facilitated by the symmetries of string theory, and one can derive explicitly the string theory modifications [58] to the low-energy effective action of section 4. They are given by automorphic functions, that are invariant functions under the discrete symmetries preserving the charge lattice [59]. They provide a complete description of the contributions of various configurations of microscopic black holes to the quantum mechanical graviton scattering processes. Their detailed properties match perfectly what is expected from a semi-classical analysis of the quantum fluctuations around microscopic black hole configurations [59]. Importantly, in the regime of small gravitational interactions, these results derived in string theory are in agreement [60] with the scattering amplitudes computed with the quantum field formalism of supergravity [33].

7. Conclusion

The remarkable relations (6) and (7) are deep and surprising connections between gravity and Yang-Mills theory in the perturbative regime where the strength of the interaction is small. They allow to evaluate and analyze many quantum gravity processes using standard quantum field theory methods. These relations reinforce the idea of a common formalism for quantum gravity and the other interactions between elementary particles. We have seen that string theory provides a consistent and powerful set-up for understanding the fundamental properties of quantum gravity. We have as well argued that at high-energy string theory provides new degrees of freedom that are necessary for a consistent formulation of the theory.

We conclude by quoting Narcissus, from the book *Narziß und Goldmund* by Hermann Hesse,

> *Man muß manchen wirklichen und manchen fiktiven Raum mathematisch berechnet und gemessen haben, ehe man als Denker an das Problem des Raumes sich wagen kann.*
>
> *A man must have measured and calculated much real and much fictitious space mathematically before he can risk facing the problem of space itself.*

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