TOWARDS AN AXIOMATIC GEOMETRY OF FUNDAMENTAL INTERACTIONS IN NONCOMMUTATIVE SPACE-TIME AT PLANCK SCALE

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Author's note*: *This review is a physics-oriented companion to the brief communication* [5] *and its formalisation* [7]*; a familiarity with at least one of those sources is advisable.

Abstract. We outline an axiomatic quantum picture unifying the four fundamental interactions; this is done by exploring a possible physical meaning of the notions, structures, and logic in a class of noncommutative geometries. We try to recognise a mathematical formalisation of such phenomena of Nature as the oriented space-time, gravity (here, dark matter and vacuum energy), Hubble's law, inflation, formation and structure of sub-atomic particles, antimatter, annihilation, CP-symmetry violation, mass and mass endowment mechanism, three lepton-neutrino matchings, spin, helicity and chirality, electric charge and electromagnetism, as well as the weak and strong interaction between particles, admissible transition mechanisms (e.g., $\mu^- \mapsto \nu_\mu + \mathsf{e}^- + \overline{\nu}_\mathsf{e}$), and decays (e.g., $\mathsf{n}^0 \mapsto \mathsf{p}^+ + \mathsf{e}^- + \overline{\nu}_\mathsf{e}$).

Our approach is based on the understanding of Physics as text which is written in the language of affine Lie algebras and associated homeo-class noncommutative structures over the space-time.

Epigraphs.

Twice earlier, twice closer.
(*From the Principle of Invariant Light Speed*)

§4. The gauge invariance.
It is known that the choice of field potentials in classical electrodynamics is not uniquely defined: the components of the 4-potential A_μ can be subjected to arbitrary *gauge* (or *gradient-type*) transformation of the form

$$A_\mu \longrightarrow A_\mu + \partial_\mu \chi, \qquad (4,1)$$

where χ is an arbitrary function of the coordinates and time (see II, §18).

Landau and Lifshitz, *Course of Theoretical Physics.* **IV**. *Quantum electrodynamics* (3rd ed., Moscow: Nauka, 1989).

Frustra fit per plura quod potest fieri per pauciora.
William of Ockham (*1285–1349*)

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Introduction. Let us try to view Physics as text whose *meaning* is Nature[1] (c.f. [11]). We now focus on its alphabet, glossary, grammar rules, and a possible location of such text where it is retrieved from, edited, and then stored back to. To pursue this goal, we perform a semantic analysis of the axioms and their mathematical implications within the algebra and calculus of structures in a class of noncommutative geometries which have been studied in [5, 7] and which were addressed earlier in [9, 14]. We attempt to formalise the geometry of such intuitive notions as the time and space that locally bears orientation and two inequivalent, homeo and diffeo, structures of topological and smooth manifold, respectively; we study the means and rules of coding massive and massless elementary particles, seeking a way to explain the annihilation of matter and antimatter or the channel for the mass-energy transmutation; we define quantum properties of particles, e.g., electric charge, spin, helicity and chirality; we explore the mechanisms of particles' transformations, reactions, and decays viewing these events as logical processing of information.

We analyse the possible physical sense of both the axioms and the operations or deductions which are admissible in the chosen setup. Still, our synthesis may be not a unique way to relate this mathematical formalism with Nature.

Remark. We attempt to identify and describe the physics which not necessarily *is*. We now describe the processes and motivate their laws which could be dominant in the early Universe only. Alternatively, it may happen that these processes are realised nowadays (or are presently registered as signals which were emitted from afar in our remote past) only under very restrictive hypotheses about the local space-time geometry, e.g., near a black hole or near its singularity.

Nevertheless, we develop the formalism in a hope that it does describe the quantum structure of the Universe at Planck scale. We are aware that a spatial resolution that fine would require us to attain the energy level 10^{19} GeV, which is currently not available; consequently, our logic should be verified by indirect evidence only. Yet this technological difficulty is not a barier for a consideration of the laws, models, and theories which adequately formalise and describe the quantum phenomena.

Our main message is this: It may be that at the Planck scale, the geometry of this world is disappointingly simple[2] because

- it does not refer to the diffeo-structure of the visible space, i.e., to its locally vector space organisation with velocity along piecewise-smooth trajectories and their length, and with smooth transition functions between charts in the atlas for that manifold; instead, the events occur in the Universe, which does not

[1] This idea is not original: it is stated in the first line of a well-known though not universally accepted literature source.

[2] This is likely if we recall how Science gradually cast away various essences such as the phlogiston (though as an abstract principle it was useful for the technology of steam engine), æther (to which we owe the radio and knot theory, recalling that W. Thomson's vortex rings preceded Bohr's planetary model of atom), or the long-range gravity force (which remains helpful for navigation in the Solar system). As a recompense, we expect that the approach which we discuss here will stimulate new insights concerning with the formation of particles and nuclei, and it will contribute to the conscious study of technologies which allow to convert the excess of mass into radiation and the excess of energy into curvature and contraction of space.

amount to the visible space, by using its much more rough homeo-structure of topological manifold with continuous transition functions, whereas the incidence relations between points along continuous paths replace the obsolete notions of length and speed;
- the Universe consists of *naught* but the homeo-class space itself and the information which it carries or is able to carry; the fact of existence, behaviour, and known forms of the interaction between particles refer to the locally available information (in particular, stored in a single point by using a local modification of the topology); the presence of gauge degrees of freedom at each point of the diffeo-class space is the manifestation of its own homeo-structure; the gauge transformations are performed pointwise, either entirely independently at different points or in no more than a (piecewise-)continuous way, whence an attempt to bind Nature with their differentiability –in order to introduce the gauge connections by taking derivatives of arbitrary functions– is an *ad hoc* assumption of the objects' description.

Indeed, let us notice that the idea of a connection in a principal fibre bundle appeals to the (existence of) structures, and to their values outside that point, which therefore requires the existence of other points. Moreover, the gauge freedom of any kind allows, in its basic formulation, pointwise-uncorrelated gauge transformations of the field of matter. Consequently, the postulate that *length* is defined and hence the distance between space-time points and between fields can be measured, giving rise to the construction of derivatives, and the postulate that the pointwise-uncorrelated values glue to a (piecewise-)smooth local section are the act of will, i.e., an *ad hoc* assumption in a description to which Nature is indifferent.[3] We remark separately that the operations which are recognised as gauge transformations but which stem from the presence and structure of space itself (e.g., local homeomorphisms of topological spaces) are at least but also no more than continuous, see sec. 2.3.1 on p. 14. We conclude that local processes can not be governed by smooth gauge theory (or only by it; in particular, gauge connection fields did not exist at the moment of the Big Bang).

Within such paradigm of quantum nihilism, the points of Nature have neither means, nor wish to learn from anywhere else what must happen inside them. If this is indeed so, the rules of Nature's behaviour are the arithmetic –comparison or addition of topologically-defined integer numbers– and the associative algebra of gluing or splitting words written consecutively in its alphabet(s), and the coding of such topological objects as walks and cycles or knots. In this review we attempt to transcribe that language, outline the principles for coding objects by words, and find out where and in which form the information is stored. We know that the text of Nature is incredibly interesting; in our efforts to read it, we have not yet advanced much in learning its grammar, and still more feebly we perceive the overall plot.

This paper is organised as follows. We first establish the reference points and formulate motivating questions. (We try to set apart the epiphenomena of description, i.e., intrinsic difficulties of a theory under the assumption that the theory itself is adequate

[3] In view of this, reading the second epigraph to this paper is particularly refreshing.

and exact, from the difficulty of encoding phenomena of Nature and rigorous definitions.) The resulting list could be timely and meaningful *per se* even if the problem of axiomatisation is not posed for the quantum geometry. However, the list hints us to seek for a possible alternative to the standard approach.

We then outline a robust and sufficiently intuitive topological picture of space and time, associating the alphabet of the Universe with affine Lie algebras; we inspect the properties of axiomatically introduced quantum geometry and recognise the objects, words, and rules whose meaning is matter, its properties, transformations, and interaction events. In particular, we shed some light on the true nature of the dark matter and vacuum energy, predict the CP-symmetry violation by viewing the laws of weak interaction as an orientation-dependent automaton, and portray the channels of free neutron's decay together with the cross-symmetric reactions.

However, we agree that the potential of our topological and combinatorial picture to explain or propose possible (dis)verifying experiments is not still the required ability for a model to predict.

1. Motivations and reference points

In this section we formulate the questions about an algebraic transcription of Nature and conversely, viewing Physics from a mathematical perspective, name the phenomena which we shall try to recognise in what follows; in the end, we shall have completed a dictionary in which axioms, definitions, structures, and logic are matched with phenomena of Nature.

1.1. What is the **space-time**? Is the set of its points *a priori* endowed with a structure of smooth manifold (see the first epigraph)? Is that smooth structure and, independently, the ($\mathbb{Z}_2$-graded) commutativity indispensable for building and for describing the world and running quantum processes, noting that those are constrained by $\mathbb{Z}$-valued invariants?

What is **gravity**? (This is not equivalent to asking what its manifestations are or, more precisely, were gravity what it is supposed to be, why would it manifest itself in precisely the way we know it?) What is the gravitational and inertial mass? What is the mass endowment mechanism?

Can vacuum exhibit gravitational effects? What could be the dark matter?

What is the space-time structure at the singularity of a black hole? Why is the entropy of a black hole proportional to the area of its event horizon, and could gravity be a thermodynamical process?

1.2. **The Big Bang.** How large was the Universe at the Big Bang? If it consisted of only one point, where and in what form did it contain

- the information about its ability to create that much matter in the course of its future evolution,
- the information (if it indeed did contain it *ab initio*) about its orientation and future asymmetries (e.g., the CP-symmetry violation and the matter-antimatter imbalance), and
- the laws of its future evolution?

How could the laws of Physics appeal to the notion of length (hence, connections) at the moment of the Big Bang ? how did those laws themselves evolve: is the present-day increase of resolution in detection at acceletators equivalent to the portrait of the early history of the Universe ?

Was the universe initially *hot* or, possibly not excluding one another, was the space-time itself contracted by a modification of its topology ? What was (and what is – at the periphery) the principle of space-time decontraction ?

1.3. Is **length** well defined and explicit in quantum processes ? Could one replace a very restrictive hypothesis of the existence of a diffeo structure of smooth manifold almost everywhere in the space-time with the homeo structure of topological manifold ? Is the causality, i.e., invariance of the cones of past and future and preservation of the incidence relations between points sufficient to build a continuous yet nowhere smooth picture of matter and interaction in which the world-lines shake but are never torn ?

Do the Heisenberg uncertainty principle *and* the observed probabilistic character of decay and interaction events imply that the Planck-scale evolution of the Universe is a topological process and is governed only by the causality ("if / only if") and incidence (or betweenness property) but without a possibility of a smooth choose of local space-time coordinates (in contrast with the first two epigraphs). Does the Universe in verity possess a full determinism[4] whereas the use of probabilistic structures in the diffeo setup is our own choice of a way to describe homeo-class Nature ?

1.4. **Dimension.** It may be that, as we improve the resolution, previously invisible directions in the space-time (their total number determining its dimension) will decompactify according to the schemes $\langle$ point $\longrightarrow$ closed space $\rangle$, hence $\langle$ world-line $\to$ tube $\to$ cilinder $\to$ (almost) plane $\rangle$. Why should the Hausdorff dimension of the resulting object(s) necessarily be integer ?

1.5. What is **energy** as an algebraic structure ? What are the admissible forms of its existence (such as the kinetic energy of particles, corpuscular character of photons, or vacuum energy) ? What is the mechanism of mass-energy transmutation $E = mc^2$?

1.6. What is **matter** ? What is the code of matter and which algebraic structures express it ? Are the synonyms allowed in the code of matter ? Where is that information stores and how is it retrieved and processed ? What is the mechanism of spontaneous decay for unstable particles ? What is the definition of point particles ? What is the geometry of particles which, in the course of lab experiments, reveal their finite size ?

1.7. What is the **antimatter** ? Should the quantitative properties of matter and antimatter be exactly equal (apart from the feature which inambiguously distinguishes between them) ? What is the mechanism of annihilation ?

Does the existence of antimatter follow from the statement that the space-time can be, or is, oriented ? Does that property imply the observed imbalance: matter prevails over antimatter in this world ?

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[4]This also reveals a link to the problem of synonyms in coding the equivalence classes that represent the same particle, see below.

What is the link between the orientation and the violation of the CP-symmetry (particles $\rightleftarrows$ antiparticles *and* Left $\rightleftarrows$ Right), which is registered for some weak processes but not for the strong interaction ?

1.8. What is the **spin** ? Why is it quantised (in units $\hbar/2$) ? Why does a considerable growth of the rest mass (approximately 0.5 MeV/c^2 for electron, 105.7 MeV/c^2 for muon, and 1776.8 MeV/c^2 for tau-lepton) not cause any increase of the inner angular momentum ($|s| = \hbar/2$ in all the three cases) ? How can massless particles carry a nonzero spin ? What is the true mass-spin correlation and does it inevitably require the (local) *Poincaré* invariance at macroscopic, non-quantum scale ?

What relates the spin of particles, their Bose or Fermi statistics, the problem of synonyms in the code of matter, and Pauli's exclusion principle ?

1.9. **Interactions.** Are the four known interactions different in their logic of processing the information about matter or do they –or some of them– manifest the unifying one ? Will the four known forces be the *only* forces available when the accelerators attain the level 10^{19} GeV ? Will the possible new ones be *gauge* theories ?

1.10. **Electromagnetism.** What is the electric charge ? How do (point) particles carry their charges ? Why is the electric charge not significant, though conserved, in other types of interaction ? Why do positive-charged protons and the neutrons form the nuclei of atoms ?

Did electromagnetism as a principle exist at the Big Bang and what was the electric charge of the Universe initially equal to ? Is the Unverse electrically neutral in the course of its evolution after the Big Bang ?

What is a mechanism for black holes to carry electric charge ?

1.11. **The weak force.** Why are there exactly three registered types of leptons: electron, muon, tau ? What are the neutrinos and the lepton-neutrino matching ? Do neutrinos have mass ? What is the mechanism of lepton-lepton transitions in the course of decays of taus and muons; what is the mechanism for neutrino oscillations ?

1.12. **The strong force.** How do positive-charged protons and the neutrons form the nuclei of atoms ?

Are the quarks real or nominal (again, arranged in two groups of three) ? What are their electric charges and spins (if those definitions are *applicable* to them) ? Why are some of the quarks unstable, and why are the six hypothesised quarks so elusive in all our attempts to detect them separately ? Is there a mechanism which prevents their existence or macroscopic observability in isolated states ?

We now attempt to understand the Universe as an information processor. While we only hypothesise about the algorithms which are actually employed to encode and process information, still we ought to define what information *is* — at least in this context. Let us temporarily accept the following heuristic definition which appeals to a common-sense idea that information must be *meaningful* and in principle *verifiable*; hence our formula is non-rigorous, possibly incomplete or self-referring, and maybe contradictory; we also remark that the quantitative *measurement* of information is a much more delicate issue (here we refer to the classical concepts of A. N. Kolmogorov, C. E. Shannon, etc.).

Definition 1. Information is a rule that inputs a message, that is, a sequence of 0's and 1's, and states a (non-)strict preference $\preccurlyeq$ or $\succcurlyeq$; by its output the rule attempts to predict the elementary event for the *next* digit of the message to be either 0 or 1. The non-strict preference $0 \succcurlyeq 1$ says that 0 is not less likely than 1 whereas $0 \preccurlyeq 1$ tells us that 1 is not less likely than 0. The act of comparison of the rule's prediction and the actually available next elementary event (possibly, itself being the first in a longer sequence) is a non-obligatory act of verification.

We accept that the processing of information is *serious*: the rule states the *same* output on the same input data whenever an input is processed twice. If the rule exhibits its readiness to change opinion for repeated input, then the elementary events 0 and 1 are *equally possible* (for example, an oracle is asked about the result of throwing a coin outside the light cone of its past). The non-strictness of preference builds the idea of uncertainty into information messages; however, if the rule states an absolute preference $0 \gg 1$ or $0 \ll 1$ for zero and unit, respectively, then the information is called *precise* (yet it may be *false*; uncertain information also can be true or false).

A sample application of such information messages is a check whether a given (i.e., contained in the input) object belongs to an encoded class of objects possessing a given property. For example, such is the coding of a triangle in a graph (see Fig. 5 on p. 26): whenever two sides are given, the rule discards all offered test edges except the one which closes the contour.

On the other hand, a sample precise message "The undetectable does exist" affirmatively states –apart from the by-product information that there exist messages in general and there are means to encode and transmit them– the existence of existence and perhaps the existence of the one who – or something which created that message; other aspects of this message's meaning are non-verifiable hence non-informative.

Summarising, *information* is (1.1) a formalised input and (1.2) a rule that establishes a preference for the next elementary event. In turn, *processing* information is (2.1) the information itself and (2.2) a second-order rule that states a preference for the choice of new rules on the basis of input rules. For example, the decay of a free neutron is processing the rule that said 'yes' whenever it was asked whether there was a neutron and which described that neutron; the processor's output states a preference that the new rules should ascertain the existence and describe the (motion of) proton, electron, and electron's antineutrino, see (1) on p. 28.

The *laws of Nature* are rules of third order, consisting of (3.1) second-order (reaction-) rules and (3.2) the rules to balance or modify the former (e.g., by prescribing the relative velocities). Such are the conservation laws for electric charge, energy, momentum, or angular momentum, etc. Notice that the laws of Nature do not refer to the formalised input (1.1) by using which one encodes the actual presence and configuration of events or particles and their properties; these laws are universal.

It is perhaps appropriate to say that the set of *fourth*-order rules for (non-)modification of the laws of Nature is the Universe itself. Indeed, should the fine-structure constant $\alpha = e^2/(\hbar c) \approx 1/137$ be varied or the dimension and topological properties of space (such as orientation or orientability) be changed, this would produce a different Universe; likewise, a slow modification of parameters in the laws of Nature (e.g., a drift of Hubble's constant in time or a change of half-life time for neutrons) would mean that the Universe itself is changing.

2. Noncommutative geometry of space-time

2.1. **The two avatars.** This Universe is a topological space. In the beginning, its topology was trivial: $\mathcal{T}_0 = \{\varnothing, \text{Universe}\}$. Nowadays it is not separable so that there are points which we can not tell one from another. Modifications of the topology $\mathcal{T}$ are also possible.[5]

Within the regions of *vacuum* where the topology allows one to distinguish between points, the Universe is endowed simultaneously with two structures: one is the homeo-class structure of topological manifold with *continuous* transition functions between coordinates (those form continuous nets on the charts); the other is the diffeo-structure of smooth manifold such that the transition functions are *smooth* and local coordinates form the smooth nets.

The Universe co-exists in its homeo and diffeo avatars. The homeo structure is the *quantum world*; it carries the information about the geometry and about the types, formation, and actual existence and states of the particles. The laws of fundamental interactions between particles retrieve and process that information, thus determining the processes that run at the quantum level. Each particle or any other object in the quantum, homeo-class geometry has a continuous world-line.

Our conscience percepts the Universe and events in it at the macroscopic level using the smooth, diffeo-class geometry, that is, by understanding of the local charts as domains in a vector space over $\mathbb{R}$ with the usual arithmetic of vectors. The notion of *length* is defined in the macroscopic world.[6] This notion allows us to measure local macroscopic distances by using rigid rods and also measure local time intervals between events by employing the postulate of invariant light speed, that is, by using derivatives of the former equipment. With the help of rigid rods and light, we introduce the macroscopic notions of instant velocity and define the nominal concept of a smooth trajectory, not referring it to any material object but only to the local properties of the smooth macroscopic space-time. The transition between macroscopic charts with smooth coordinates in the diffeo-class space-time are governed by the Lorentz transformations. The topology of macroscopic space-time outside particles and black hole singularities is induced by the (indefinite) metric in inertial reference frames.

The tautological mapping from the quantum, homeo-class world to the macroscopic, diffeo-class realisation of the Universe is continuous but not a homeomorphism; by construction, it can not be an isometry. Under this tautological mapping, the information which is realised by the quantum geometry takes the shape of particles in the macroscopic world; however, a part of this information is lost along the way due to the introduction of length (more precisely, of Lorentz' interval): there are topologically-nontrivial quantum objects –in fact, a whole dimension– which acquire zero visible size in all reference frames.

On top of that, because the composition of (1) local homeomorphisms from the standard domains to the charts of the homeo-class topological manifold with (2) the tautological mapping is only continuous at all points of the Universe, the images of

[5] We claim that exclusions of sets from the list $\mathcal{T}$ and re-inclusions of the information about such sets provide the mass endowment mechanism and formation of the black holes' singularities, see below.

[6] By convention, a length scale is *macroscopic* if the typical distances considerably exceed the electric-charge diameter of proton, which is approximately $1\ \mathsf{fm} = 10^{-15}\ \mathsf{m}$.

points and point particles in its visible, diffeo-class realisation are observed by us as if they are in a perpetual inexplicable "motion." Namely, as it often happens with legal documents, no other rights may be derived from the statement that the composition is continuous: the pledge is to take points from nested sets in the atlas $\mathcal{T}$ to near-by points as we see them, but the continuous mapping does not presume that we, upon our own initiative, shall apply our notion of length to some continuous curves connecting those images. In effect, inertial trajectories of material objects in the quantum world are continuous but nowhere differentiable.[7] The visible world-lines are at most $(c, 1)$-Lipschitz, where c is the speed of light and the power 1 states that no material object is allowed to run out of the light cone of its future.

Example 2.1. Suppose that we know (setting aside all the subtleties related to the act of measurement) that a quantum object –e.g., a marked point of it where all the mass or all the charge is contained– is located now at a given point and moreover, it does not move with respect to other points according to the incidence relations between points in a separable topology. Nevertheless, we may not know its visible instant velocity because that notion refers to the limit procedure in a vector space and hence is not applicable.

Example 2.2. Likewise, choose an inertial reference frame and consider a situation when a domain in quantum space is homeomorphic to a domain within a crystal structure [3, 4]. Suppose that a vertex of the lattice decides to visit its neighbour and thus goes along the edge connecting them. Not only its visible initial location was non-constant in time and the initial instant velocity undefined, but this will remain so at all points of the continuous, nowhere-differentiable image of the trajectory along the edge; the journey will end at an unpredictable location of the endpoint with undefined terminal velocity. We conclude, referring again to 'A Hard Day's Night,' that it is impossible to tell which of the two worlds, topological homeo or smooth diffeo, is straight and which is shaking.

For the same reason, provided that we postulate a point particle's *visible* instant velocity (irrespective of its actually on-going displacement with respect to the topologically-neighboured points in the quantum world), we may not determine at which point of the visible macroscopic space it is located. (Let us remark that the above examples and reasonings are not applicable to the propagation of light which can not stay at rest with respect to the incidence relations.) The balance of resolution for the location of a material quantum object in the smooth space at a given time and for its momentum is determined by the Heisenberg uncertainty principle.[8] Simultaneously, the propagation of a point particle from a given point to a given endpoint along an *a priori* unknown continuous trajectory in the visible, diffeo-class world is the cornerstone of the concept of Feynman's path integral.

We now propose to abandon the futile attempts to measure or approximate the undefined notions but study the interactions between quantum objects by referring

[7] This creates the classical antagonism between Red and White: namely, Norm is firm, straight, and always all right while Shake is indecisive and trembling. Only we cannot tell who is who in this Universe.

[8] Note that we may not track the behaviour of "empty" points of the quantum space if they are not referred to by any material object located there; consequently, we do not attempt to introduce a "temperature" of the vacuum.

their laws to the homeo-class geometry of the Universe. Let us remember that the difficulties and uncertainties which we gain –when measuring length and calculating derivatives such as the velocity– in a description of the quantum processes do not stem from their true nature. Integrating empirically the laws of its evolution, the Universe stays, and will stay forever indifferent to the fact that we can not grasp all its details at once, since we ourselves first proclaimed our intention to take proportions with respect to the standard metre instead of inspecting the topological invariants of phenomena.

Corollary 1. The processes in the quantum, homeo-class (locally) separable topological manifold without length can not be adequately described by (the geometry of partial) differential equations (c.f. [6, 13]). On the other hand, the construction of σ-algebras associates the *measure* in its true sense with sets of points but not with distances between points; consequently, *integral* equations could be more relevant.

2.2. **The time phenomenon.** There are at least two ways to understand what the *time* is in context of a paradoxal observation by our conscience that everything in this world is staying perpetually in the *present*.

A realist approach to the notion of time postulates the existence of a full-right uncompactified dimension with a reasonable topology of the resulting space-time. One then operates with the count of time by using the invariant Lorentz interval, light cones of the past and future, and world-lines of material objects. An inconvenience of this approach is that, in order to maintain the everlasting presence, the visible world must unceasingly glide along the time direction, i.e., to keep in the same place, it takes all the running one can do. Note that under Lorentz' transformations the local observer's time can be bent towards another observer's space and *vice versa* but in earnest the time can not be swapped with any spatial direction.

The concept of a $(3+1)$-dimensional smooth or topological manifold into which the time is incorporated *a priori* contains the following logical difficulty. An infinitely-stretching absolutely empty, flat Minkowski space-time $\bigl(\mathbb{R}\times\mathbb{R}^3, (+---)\bigr)$ without a single object in it would exist *forever*. In our opinion, there is no time at all in that empty world: the cups, tea, and bread-and-butter always remain the same, so it is always six o'clock. Nobody counts to the Time hence time does not count.

Definition 2. We accept that the time is a count of reconfiguration events in this Universe; such events are, for example,[9] the reconfigurations of geometry (i.e., an act of modification in the topology) or the operation of an algorithm that transfers information over points, creating an event of output statement by processing the local configuration of the Universe in its input (such is the propagation of light).

Thus, events create time. Events which do not reconfigure the Universe (e.g., a correct statement that for a topologically-admissible arc connecting point $\underline{0}$ to point $\underline{1}$ there is a null path running from $\underline{0}$ to $\underline{1}$ and then back again along the same arc) *do not* express the count of time (although a *verification* of such statement by using light signals does take and hence creates time).

[9]Were the Universe truly smooth, Poisson, and possess the Hamiltonian functional, then the time by definition will be taking the Poisson bracket with such master-functional of the current state; a weakened and much more likely formulation is the generation of time by events of evaluating binary operations at locally defined Hamiltonian functionals that correspond to separate particles, c.f. [7].

The notions of recorded past and expected future are derived from the relation of order in the count of events by an appointed observer; let us remember that an opinion of another observer about the order of events could be different.

Example 2.3. Consider the reconfiguration of the Universe produced by a trip of Chapeau Rouge from point $\underline{0}$ to point $\underline{1}$ along a continuous arc connecting them in a coordinate chart of the homeo-realisation of the Universe. This amounts to the input information that the two endpoints, the arc, and Chapeau Rouge exist, that the available choice of topology confirms that the path is continuous, and to the work of the algorithm the negates the already passed points and thus prescribes the admissible direction to go further.

In absence of length and in absence of any devices at the observer's disposal, the time is discrete: it is counted by the events (1) Chapeau Rouge is at the starting point; (2) Chapeau Rouge has reached the endpoint.

The observer can grind the time scale by recursively installing the intermediate checkpoints somewhere in between the points which are already marked; this is done by using the incidence relation for points on the continuous path and does not refer to the notion of length (in fact, it refers to the definition of real numbers by using 0, 1, addition, and bisection). The limiting procedure makes the count of time continuous.

It is the postulate of invariant light speed which endows the Universe with its local smooth structure ("twice earlier $\Longleftrightarrow$ twice closer"). The light automaton is programmed to choose the next point by processing the information about earlier visited points and creates an event of specific type; the principle is that all observers accept its performance identically. By using the bisection method, we first mark the midpoint $\underline{{}^1\!/_2}$ on the chosen curve and replicate the automaton $\underline{0} \to \underline{1}$ to the automata $\underline{0} \to \underline{{}^1\!/_2}$ and $\underline{{}^1\!/_2} \to \underline{1}$; then we declare that the old automaton counts the unit step of time and each of the new automata counts one half. The recursive process and the limiting procedure create the smooth structure of space-time for a given observer.[10]

The inconvenience is that this smooth structure is not applicable to material objects which are known to travel slower than light; in order to monitor the steady progress of Chapeau Rouge on her way from $\underline{0}$ to $\underline{1}$, one must use as many light signals as there are checkpoints installed along the path. Even if the *energy* emitted by the new, "shorter-range" automata drops at the moment of each replication, the total energy which one has to spend in the continuous limit is either null or infinite; the first option is useless because it does not communicate any information to the observer; the second option is not impossible if the Universe is infinite and the observer agrees to waste a finite fraction of this world, still it is impractical.

In the next section we introduce a possible local topological structure of the homeo-class realisations of the Universe. If one feels it necessary to multiply dimensions, then we advise to let a macroscopic observer view the construction from a chosen inertial

[10]To use light as the pacemaker of the clock, we ought to describe first what a photon is; to do that we operate with the homeo-class geometry. We also notice that *before* the emission of the first photon in history of the Universe (or before creation of any other massless particles which travel with the same invariant speed c), the time had been counted by using events of other origin; we argue that such events were the reconfigurations in the topology $\mathcal{T}$.

frame; the time direction is then locally decoupled to the real line $\mathbb{R}$. At the end of the next section, the structure of the macroscopic images of domains under the tautological mapping from homeo to diffeo is then recalculated to all other inertial reference frames by using Lorentz' transformations. Let us only remark that the "smooth time" parameter is introduced in the diffeo-class world in order to legalise the limiting procedures such as the correlation of arbitrarily small length and the speed of light in the local vector-space organisation of space.

2.3. **Local configuration of quantum space.** Now we introduce the local topological configuration of empty quantum space, that is, *vacuum* away from the singularities of black holes. In technical terms, we define the admissible local structure by taking "as is" or via self-similar continuous limit the lattice of affine Lie algebra (primarily using the root systems A_3, B_3, or C_3 of simple complex Lie algebras) and thus consider a truncated Kaluza–Klein model without the Minkowski dimension of Newtonian time but instead, with a topology brought in by hand (though it is equivalent to a standard one for each continuous limit); we then analyse the origins of Hubble's law.

To describe a domain in quantum, or *quantised*, space we first consider Euclidean space $\mathbb{E}^3$ containing the affine lattice generated by the irreducible root system A_3, B_3, or C_3 (see Remark 2.5 on p. 15). Let us denote by $\vec{x}_i \equiv \vec{x}_i^{\pm 1}$ the generators of the lattice at hand and by $\vec{x}_i^{-1}$ their inverses (so that the paths $\underrightarrow{\vec{x}_i \cdot \vec{x}_i^{-1}} = \underrightarrow{\vec{x}_i^{-1} \cdot \vec{x}_i} = 1$ end at the point where they start). Each lattice determines the tiling of space $\mathbb{E}^3$, its vertices and edges constituting the 1-skeleton of the CW-complex with trivial topology (see Remark 2.7 on p. 18). We let a finite domain in $\mathbb{E}^3$ with a given configuration of vertices and the adjacency table of the lattice be the *spatial component* of a prototype domain in the discrete quantum space.

Second, we take the product $\mathbb{E}^3 \times \mathbb{E}^2$ of space with a two-plane into which we place the circle $\mathbb{S}^1$ passing around the origin. Viewing the circle as an oriented one-dimensional topological manifold, we create an extra, compactified dimension in the local quantum geometry. Namely, to each vertex of the prototype domain we attach the *tadpole* $\mathbb{S}^1$, i.e., the edge that starts and ends at the same point and loops in the extra dimension (outside the old $\mathbb{E}^3$). By convention, we denote by $\mathbb{S}^1 \equiv \mathbb{S}^{+1}$ the tadpole walked counterclockwise with respect to the standard orientation of $\mathbb{E}^2$ and by $\mathbb{S}^{-1}$ the reverse, clockwise cycle.

Remark 2.1. Under the tautological mapping of the quantum world to the diffeo-class visible world, the tadpoles are assigned zero length because the distance between their start- and endpoints vanishes for each of them.[11] We conclude that the entire compactified dimension is invisible to us; this is why the tautological mapping between the homeo- and diffeo-realisations of the Universe is not bijective: it compresses one extra dimension at each point to a null vector.[12]

[11]We note that the notion of length *is* applicable to the generators $\mathbb{S}^{\pm 1}$ of such null vectors but it is not applicable to the edges $\vec{x}_i^{\pm 1}$ in space: their length is undefined because the homeomorphisms from domains in $\mathbb{E}^3$ containing the lattice to the spatial counterparts of the prototype domains are not fixed but can change with time.

[12]We also notice that the generators $\mathbb{S}^{\pm 1}$ encode nontrivial walks in quantum space but produce no visible path which would leave a single point in the macroscopic world; we claim that the contours $\mathbb{S}^{\pm 1}$ determine the electric charge $\pm e$, see sec. 3.3.2.

Namely, passing to the additive notation $\bigl((\vec{\mathrm{x}}_i, \vec{\mathrm{d}}), \pm\bigr)$ instead of the multiplicative alphabet $\bigl((\vec{\mathrm{x}}_i^{\pm 1}, \mathbb{S}^{\pm 1}), \cdot\bigr)$, that is, viewing the letters as vectors in Euclidean space but not as the shift operators and introducing the null vector $\vec{\mathrm{d}}$, we recover the standard description of the affine basis for the Kac–Moody algebra at hand; clearly, the length of the null vector equals zero. Recall further that the circle $\mathbb{S}^1$ is the total space in a double cover over the real projective line $\mathbb{RP}^1 \simeq \mathbb{S}^1/{\sim}$; one full rotation $\mathbb{S}^{+1}$ corresponds[13] to running along the projective line twice: $\mathbb{S}^1 = \overrightarrow{\mathrm{tt}}$ and $\mathbb{S}^{-1} = \bigl(\overrightarrow{\mathrm{tt}}\bigr)^{-1}$; the double cover over $\mathbb{RP}^1$ is then responsible for the familiar coefficient '2' in front of the null vector $\vec{\mathrm{d}}$.

Remark 2.2. The vertices of the CW-complex are the *quanta* of space; there is a deep logical motivation for their existence. Namely, by assembling to one vertex a continuum of physical points within a domain which is dual to the set of neighbouring vertices in the lattice, Nature replaces the *continuous* adjacency table between points to a *finite*, lattice-dependent table so that there are only finitely many neighbours of each vertex and hence a finite local configuration of information channels.

In conclusion, space is continuous but the Universe operates with quantum phenomena in it, thus achieving a great economy in the information processing.

Remark 2.3. The tadpole $\mathbb{S}^1$ at a vertex of quantum space is an indexed union $\bigcup_{i\in\mathcal{I}} \mathbb{S}^1_i$ of tadpoles referred by i to an indexing set $\mathcal{I}$ of points in the quantum domain which is marked by the vertex. Typically, this set is at least countable, $\mathcal{I} \supseteq \mathbb{Z}$; one could view it as an enumerated set of binary approximations for points in that domain (here we use the auxiliary metric in $\mathbb{E}^3$); we emphasize that by choosing the indexing set in this way we endow it with *order*, c.f. sec. 3.3.3.

This convention allows us to handle infinitely many tadpoles attached to an everywhere dense set of the quantum domain by ascribing a different statistics to a unique tadpole which is attached to the vertex which marks that domain.

Convention. We postulate that the spatial edges $\vec{\mathrm{x}}_i^{\pm 1}$ of the lattice are *fermionic* so that no such edge can be walked twice in the same direction by one path; a path can walk twice along the same edge only in the opposite directions. Note that different paths can walk independently in the same direction along a common edge; we also notice that a path can run many times through a vertex, approaching it each time by a different edge in its adjacency table.

Unlike it is with spatial edges, the tadpoles $\mathbb{S}^{\pm 1}$ attached to the vertices are *bosonic* so that paths can rotate on these caroussels any finite number of times in any direction (which does not really matter because the overall difference $\sharp\mathbb{S}^1 - \sharp\mathbb{S}^{-1}$ of positive and negative rotation numbers is constrained by the value of electric charge of the particle encoded by the path). However, let us remember that in earnest we are dealing with ordered infinite sets $\{\mathbb{S}^{\pm 1}_m,\ m \in \mathbb{Z} \subseteq \mathcal{I}\}$ of *fermionic* tadpoles brought to the marker of a quantum domain; in the continuous limit of a quantum tiling, this set spreads over the domain — one fermionic tadpole per each indexed point.

[13] We introduce a separate notation $\vec{\mathrm{t}}$ for one rotation along the projective line anticipating its future application in the description of "building blocks" for strong interaction (see sec. 4.2) and also its possible use in the study of the quantum Hall effect.

So, let us recall that the part of a lattice in a domain of $\mathbb{E}^3$, with tadpole attached to each vertex, is *discrete*. We say that the original fermionic lattice with bosonic tadpoles is the *quantum space*; in what follows we formalise the geometry of elementary particles in terms of the *alphabet* $\mathfrak{A} = \bigl((\vec{\mathrm{x}}_i^{\pm 1}, \mathbb{S}^{\pm 1}), \cdot\bigr)$.

The standard bisection technique (see sec. 2.2) allows us to convert the discrete tiling to its continuous limit in which the topology is inherited from the adjacency table of the affine lattice (the neighbourhoods are the duals of adjacent vertices' configuration in the spatial, $\mathbb{E}^3$-tiling component of the CW-complex); the limit topology is locally equivalent to the product topology for $\mathbb{S}^1$ and Euclidean space $\mathbb{E}^3$.

Definition 3. The self-similar limit of the discrete structure in a domain of quantum space is a *domain* in the homeo-class realisation of the Universe.

Remark 2.4. The introduction of a continuous field of fermionic lattice generators $\vec{\mathrm{x}}_i^{\pm 1}$ and fermionic loops $\mathbb{S}^{\pm 1}$ or $\vec{\mathrm{t}}^{\pm 1}$ over *each* point of continuous space, which we have performed here in full detail, is the homeo-class analog of the *noncommutative tangent space* over the smooth visible realisation of the Universe, see [7].

Quantum space is discontinuous; in sec. 2.2 we argued that a verification of the continuity for its self-similar limit requires the expense of infinite energy whenever one attempts to monitor a steady motion of a material object travelling slower than the speed of light and for that purpose encodes the object's path by the alphabet $\mathfrak{A}_\infty = \bigl((\frac{1}{2^n}\vec{\mathrm{x}}_i^{\pm 1}, \mathbb{S}^{\pm 1};\ n \in \mathbb{N} \cup \{0\}), \cdot\bigr)$. However, a motivation why the limit should nevertheless be studied –and is more than a mathematical formality– is as follows. Namely, a *continuous* coding of points in space by using binary arithmetic permits us to consider continuous paths –in particular, closed contours,– not referring them to a specific lattice. Indeed, our ability to describe and handle such contours does not imply that any material object is actually transported along those paths; hence energy is not spent but the drawn figures, and homotopies of these images in space, do encode information: a generic continuous path is an infinitely-long cyclic word written by using infinitely-short letters of the alphabet $\mathfrak{A}_\infty$. The massless chargeless contours propagate freely in homeo-class domains until a very rare event of their disruption and weak interaction with other material objects. However, this is only a part of the story.

2.3.1. *The $U(1) \times SU(2)$-picture.* First, let us notice that there is no marked origin in the affine lattice and therefore it acts on itself by finite shifts. Note further that this action is topological: it appeals to the incidence relations between vertices but not to the smooth, local vector-space organisation of $\mathbb{E}^3$.

Having placed the affine lattice in $\mathbb{E}^3$, one could –by an act of will to which Nature is indifferent– extend the algebra of finite shifts to the space of homeomorphisms of $\mathbb{E}^3$, i.e., the local action of space upon itself by a continuous field of translations. Moreover, by compactifying space to $\mathbb{E}^3 \cup \{\mathrm{pt}\} \simeq \mathbb{S}^3$, one extends this action to homeomorphisms of the three-sphere. By yet another misleading isomorphism $\mathbb{S}^3 \simeq SU(2)$ –which is given, e.g., by the Pauli matrices– one is tempted to conclude that

(1) the complex field $\mathbb{C}$ is immanent to static geometry of the Universe, and
(2) the freedom of appointing for reference point any vertex in the affine lattice, now realised as a set of points inside $SU(2)$, means the introduction of the $SU(2)$-principal fibre bundle over the space-time.

Yet even more: though the pseudogroup of local homeomorphisms of space states that the field of pointwise-defined shifts is continuous, it is postulated that this deformation field is smooth, hence there exist derivatives of local sections for the principal fibre bundle. This pile of *ad hoc* conventions delimits the smooth complex $SU(2)$-gauge theory of weak interaction.

Likewise, each tadpole's circle $\mathbb{S}^1$ carries the gauge freedom of marking a starting (hence, end-) point on it and also it can be subjected to an arbitrary homeomorphism (not necessarily a diffeomorphism), which leaves the tadpoles $\mathbb{S}^{\pm1}$ intact. The choice of marked points is made *pointwise* at vertices of the lattice (or at all points of continuous space if we deal with the limit) — even without any idea of continuity (or smoothness, superimposed to it). Now we note another misleading isomorphism $\mathbb{S}^1 \simeq U(1)$, which also tempts one to introduce complex numbers in the static quantum geometry.

Summarising, we see that electroweak phenomena are quantum space in disguise.

2.3.2. *Hubble's law.* Second, let us recall that there are exactly three canonical tilings of Euclidean space.

Remark 2.5. The three irreducible tilings of space are determined by simple root systems A_3, B_3, and C_3. They have equal legal rights in the geometric construction, still we believe that the tetrahedral tiling which corresponds to A_3 dominates over the two others whenever one is concerned with the symmetry and stability of particles whose contours are encoded by words written in these root systems' alphabets (see sec. 3). Thus, more symmetric particles are more stable.

We recall that the adjacency tables for vertices are different for the three irreducible lattices in $\mathbb{E}^3$ so that the local configurations of information channels between points in the continuous limits are also different; the three continuous versions of one space differ by the algorithms of processing locally available information.[14] However, the limit topologies are equivalent in a sense that a continuous path in one picture stays continuous in any of the other two; the transliteration of a continuous path then amounts to a *second order phase transition* when the object stays identically the same but the underlying crystal structure changes.[15]

In the sequel, we prefer to operate locally on the affine lattice A_3 yet we allow a formal union of the three irreducible alphabets in the fibre of the noncommutative tangent bundle over each point of space. We view the irreducibility as the mechanism which holds space from slicing to lower-dimensional components; because of that, we shall not consider the reducible cases $A_1 \oplus A_1 \oplus A_1$, $A_1 \oplus A_2$, $A_1 \oplus B_2$, and $A_1 \oplus G_2$. We also emphasize that we always preview a possibility of taking the continuous limit in mathematical reasonings but we let the space be quantised by the edges of the graph, i.e., by the 1-skeleton of the CW-complex.

[14] Recall that the two gentlemen of Verona could embark and sail to Milan with the morning tide; alternatively, they could take a train, fly in an airplane, or go by car. Their route organisation would have been different in these four cases, yet the starting- and end-points coincide; the four paths are integrated by rotation of screw or wheel.

[15] We expect that the transliterations –from one alphabet to another– of cyclic words encoding the contours whose meaning is a chargeless spin-$\frac{1}{2}\hbar$ particle explains the known neutrino oscillation.

Viewing the world as it is (e.g., compared with the multiplicative structures in [7]), we have to admit that a perfectly ordered life inside a Kac–Moody algebra is an inachievable ideal. In practice, the 1-skeleton of the CW-complex experiences an everlasting reconstruction; this is why up to this moment we have not described the attachment algorithm or transition mappings between overlapping quantum domains; they just attach as graphs and the verity is that the CW-complex is globally defined — it *is* space in which the Universe exists.

A possible mechanism of the perpetual modification in the graph's local topology (but not in the triviality of topology of the CW-complex) is that Natura abhorret a vacuo. In its zeal to shake off its quantum discontinuity, Nature does attempt to perform the infinite bisection and construct the complete real line $\mathbb{R}$ by using binary arithmetic. Let us recall that such recursive procedure replicates one unit-time light automaton to two automata plugged consecutively, one after another. But Nature unceasingly replicates each light automaton with its two copies that are *identical* to their sample. This leads to the observed proper elongation of space.[16]

Namely, within each fixed half-time[17] on-average one half of the actually available edges split in two new edges. (We remark that this division does not happen with the *contracted* edges, see next section; on the same grounds it is the edges but not vertices that split, for the latter could in fact be a superposition of many vertices according to the record of past modifications in local topology.) Each event of edge splitting creates a new vertex –the midpoint– and fills in the adjacency table for it, connecting it by one edge with all vertices in the cells delimited by the splitting edge in their faces.

Also, a tadpole is attached to the new vertex within the compactified dimension. We recall that the vertices label quantum domains in space so that the graph's adjacency table configures the domains' neighbours. We now note that the process of spontaneous edges' splittings roots in the conventional round-up $[n-\frac{1}{2}, n+\frac{1}{2}) \mapsto n$: the edge's midpoint is referred to *one* of the edge's endpoints — hence, the midpoint's fermionic tadpole is communicated to that endpoint. The splitting goes as follows: in terms of the order in $\mathbb{Q}$ along the splitting edge $(n-1, n)$, its midpoint $\{n-\frac{1}{2}\}$ detaches from $\{n\}$, proclaiming the existence of a separate quantum domain $[n-\frac{3}{4}, n-\frac{1}{4})$ which becomes adjacent with $[n-\frac{3}{2}, n-\frac{3}{4})$ and $[n-\frac{1}{4}, n+\frac{1}{2})$; the round-up demarkation reproduces twofold at $\{n-\frac{3}{4}\}$ and $\{n-\frac{1}{4}\}$. The marker $\{n-\frac{1}{2}\}$ of the new domain grabs –and endows with order the set of– fermionic tadpoles attached to all the points $\{n-\frac{3}{4}+\frac{m}{2^\ell}\}$ which (by the values of $m, \ell \in \mathbb{N} \cup \{0\}$) locally get into the bounds $[n-\frac{3}{4}, n-\frac{1}{4})$.

But because the light automata remain the same for the first and second fragments of the edge, each of them counts the propagation of light signal along each new edge as a unit-time event. Consequently, not only the Universe grows at its periphery, but a trip between distant objects all across the Universe takes more and more time.

[16]Notice that a release of energy in the course of edge decontractions (see sec. 2.4) is compensated with a simultaneous increase of the volume of continuous space so that the energy density remains constant; then, a part of this energy is being absorbed by the black holes or is radiated to the spatial infinity. A simultaneous release of ever-growing amount of energy per unit-time at the outer periphery of decontracting Universe (see next section) does not burn the objects inside it but it instead cools down to the present 2.725° K of the cosmic microwave background radiation.

[17]This time interval is counted by the local billiard clock –the edge itself– that sends light signals to and fro the edge; this parameter can vary as the Universe grows older.

Corollary 2 (The Hubble law)**.** The Doppler-shift-measured velocity v at which distant material objects, locally staying at rest with respect to physical points, i.e., with respect to the incidence relations between points in quantum space, recede from each other is *directly proportional* to the proper distance D between these objects:

$$v = H_0 \cdot D,$$

where H_0 is the Hubble constant (now it approximately equals 74.3 ± 2.1 $(\mathsf{km/s})/\mathsf{Mpc}$).

Notice that the picture is uniform with respect to all observers associated with such objects anywhere in space.[18]

Thus, Hubble's law testifies steady self-generation of space due to which Cosmos obeys the principle "twice farther, twice faster" at sufficiently large scale. We conclude that we do hear the process of space expansion in the form of the cosmic microwave background radiation; we thus predict that the 1.873 mm-signal can not be altogether shielded by any macroscopic medium.

2.4. **The mass.** The crucial idea in our description of the geometry of vacuum –which is not inhabited yet by any particles– is that contractions of edges in the graph are allowed (but highly not recommended unless possible consequences are fully understood). We emphasize that this does not require stretching, pulling, compressing, or any other forms of physical activity — an ordinary accountant with pencil, eraser, and access to the book $\mathcal{T}$ with topology of the Universe can accomplish more epic deeds in the course of one reaction than Heracles did in his entire life.

Definition 4. The *contraction* of an edge is a declaration that its endpoints merge and there remains nothing in between (i.e., a tadpole is not formed); the respective ordered sets of fermionic tadpoles $\mathbb{S}$ attached to the merging endpoints unite, preserving the tadpoles' directions and their ordering (so that a path in positive or negative direction along either of the bosonic understanding for the old tadpoles becomes the respectively directed path on the new one). The *decontraction* of a previously contracted edge is its restoration in between its endpoints which become no longer coinciding, and the splitting of the indexed ordered tadpole sets between the two endpoints.

Remark 2.6. A contraction of edges in the 1-skeleton of the CW-complex can force the formation of tadpoles from remaining edges, see Fig. 3 for an example of two contractions which distinguish between Left and Right, thus leading to the CP-symmetry violation in weak processes.

Let us also notice that the on-going splitting of edges, which is responsible for the Hubble law, is a random decontraction process spread over quantum space.

Let us inspect how the concept of Riemann curvature tensor works in the noncommutative setup when one transports an edge in the CW-complex's 1-skeleton along a contour starting at a vertex formed by contracting an edge (see Fig. 1 on the cubic lattice). Namely, let the edge ab be contracted; consider the lattice element $\vec{\mathrm{z}}$. First, transport its starting point a along the contour $\underrightarrow{\vec{\mathrm{x}}\vec{\mathrm{y}}\vec{\mathrm{x}}^{-1}\vec{\mathrm{y}}^{-1}}$ and *then* step along $\vec{\mathrm{z}}$; the

[18] A relative motion of the Milky Way with respect to the underlying quantum space structure is observed by the detection of Doppler's shift in the relic radiation at certain direction and its antipode.

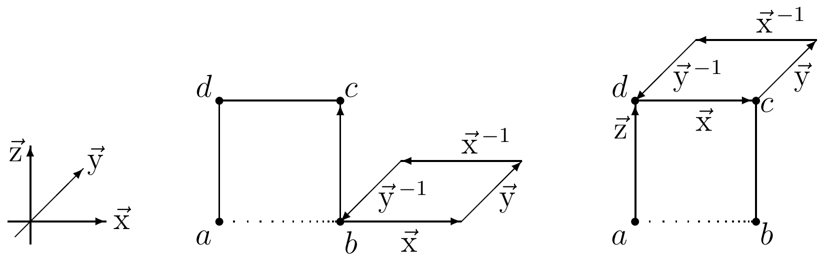

FIGURE 1. The curvature mechanism.

walk's endpoint is c. However, by walking the route $\overrightarrow{\vec{z}\vec{x}\vec{y}\vec{x}^{-1}\vec{y}^{-1}}$ and thus transporting the endpoint along the chosen contour, one reaches the vertex d instead of c. By definition, a path connecting c to d is the value $R_a(\vec{x}, \vec{y})\,\vec{z}$ at $\vec{z}$ of the quantum curvature operator for the path determined by the ordered pair $(\vec{x}, \vec{y})$ at the point a.

Convention. In this text we postulate, not deriving the mass-energy balance equation $E = mc^2$ from the underlying geometric mechanism, that a *presence* of a contracted edge is seen as mass, whereas a time-generating event of reconfiguration absorbs energy –creating mass– by contracting an edge and releases that energy at the endpoints in the course of its decontraction; this is the mass-energy correlation mechanism. (Note only that one may not measure the stored energy as "force×distance" and thus introduce a stress of the lattice because length is undefined on it).

Corollary 3. In absence of visible matter and energy, the vacuum can be curved and cause the force of gravity.

We expect that the *dark matter* is the configuration field of space contractions along its subsets; in fact, massive but invisible dark matter is not matter at all.

Let us define the *entropy* S of a given contraction of edges as minus the number of vertices — which themselves are not attached to the contracting edges but which neighbour via an edge with vertices that merge. Basically, this entropy of a contractions configuration in space is the topologically-dual to *area* (here, the number of faces for the nearest surface that encapsulates the contracted edges).

For example, a contraction of one edge of the square lattice in $\mathbb{E}^2$ produces a snowflake so that $S = -6$, making $2 \times (-6)$ for two distant contractions. However, let us notice that the entropy of two consecutive edges for that tiling equals -8 and is equal to -7 for a corner. In view of this, a reconfiguration of contractions in a finite volume of the graph could be a thermodynamical process.

Conjecture 4. Initially, the entire space of the Universe was contracted to one point so that every tiling of it by the 1-skeleton of the CW-complex was either that vertex or the vertex with tadpole attached to it.

Simultaneously, we expect that space is topologically trivial so that all its possible tilings may not contain extra edges which create shortcuts between distant cells.

Remark 2.7. Numeric experiments [1] reveal the following property which the CW-complex in $\mathbb{E}^3$ gains in the course of bisection –i.e., making the tiling finer– under an extra *ad hoc* assumption that the cells can be glued, orientation-preserving, along prescribed pairs of faces not necessarily to their true neighbours but possibly to sufficiently remote cells. This creates a possibility to obtain a topologically nontrivial CW-complex which nominally fills in $\mathbb{E}^3$ but such that the local density of genus can be positive. The probability of reconfigurations was postulated to drop exponentially with increase of the $\mathbb{N}$-valued distance between cells.

Then the fundamental solution of the usual heat equation –or the square mean deviation of random walks– was calculated by using a natural convention that the dissipating medium (e.g., smoke) or the random walks' endpoints spread freely through the faces of reconfigured tiling. The effective dimension was then determined from the rapidity of dissipation, and the modelling was repeated a suitable number of times.

Numeric experiment has shown that, as the sides of elementary domains become smaller but the effective distance, at which the probability of faces' reattachment drops $\exp(1)$ times, is kept constant, the effective dimension of (3+1)-dimensional combination of space and time drops from four to *exactly two* in the continuous limit.

3. Noncommutative geometry of particles

3.1. **Formation and structure of sub-atomic particles.** Suppose that the graph, i.e., 1-skeleton of the CW-complex is given and $\mathbb{S}^{\pm1}$, $\vec{\mathrm{x}}_i^{\pm1}$ is the alphabet associated with its vertices; we note that it can be vertex-dependent due to the on-going spontaneous splitting of edges, which creates the irregularity of the graph's structure and its possible local deviation from the regular tiling given by root systems.Then every word in the alphabet(s) determines the path along the edges, starting at a given point; we shall consider the words of length zero and their synonyms separately at the end of this section.

In what follows, we view paths as equivalence classes of walks because of a possibility to insert, wherever possible, trivial paths $\langle a\cdot a^{-1}\rangle$ along the spatial component of the 1-skeleton or a trivial word $\langle \mathbb{S}^1\cdot\mathbb{S}^{-1}\rangle = \langle 1\rangle = \langle \mathbb{S}^{-1}\cdot\mathbb{S}^1\rangle$ by using the tadpole attached to every vertex. We only recall that the spatial edges of the graph are fermionic and thus may be passed at most once in each of the two directions; this eliminates the risk of infinite loops.

We note that the paths (or walks) of positive proper length[19] can happen to be closed, i.e., end at their starting point but not retract to it if one pulls by both ends of a thread that has been unrolled along the edges of the path. Notice further that such cycles can equivalently start and end at any other vertex along the contour; thus, the words encoding them are *cyclic-invariant* (see [5, 7] and [9]).

There are several mechanisms for a given path to be closed (apart from being a tadpole $\mathbb{S}^{\pm1}$ hence closed by definition). First, there is a *glossary*, i.e., a list of cyclic-invariant words (more precisely, a point-dependent *gallery* of drawn contours); we shall quote from that source in sec. 3.3 and sec. 4. Second, there could be an additional list of (formal sums of) paths which are themselves not closed but which link to a contour whenever attached consecutively in suitable order, see sec. 4.2. Thirdly, one can proclaim that a given path is closed by manually contracting a set of edges between its loose ends; this may require a considerable or infinite energy (recall that the topology

[19] The proper length of a path is the minimal number of edges in this path and in all its synonyms that may differ from it by synonyms of zero-length word inserted at any vertex along the way.

of the CW-complex is trivial and one may not return to the starting point by walking in one direction and still coming back around the entire Universe).

Next, there is a mechanism that generates cycles in the course of decontraction of edges. Namely, consider the synonyms $\langle 1\rangle = \langle \underrightarrow{\vec{\mathrm{x}}_i\vec{\mathrm{x}}_i^{-1}}\rangle$ and suppose that the vertex which the null path does not leave is a pair of vertices connected by a contracted edge, see Fig. 2. The formation of cycle is completed by matching the direction in which the new,

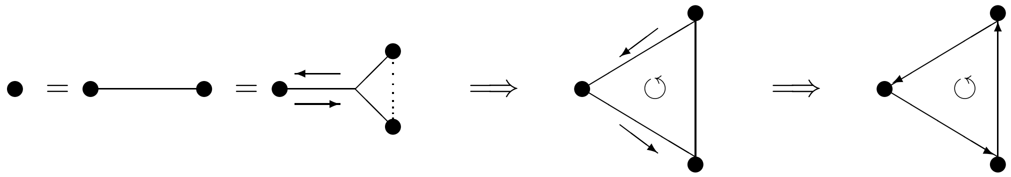

FIGURE 2. Creation of a contour.

decontracted edge is passed with the new face's orientation induced from the oriented CW-complex; note that energy is released in this process.

Finally, the energy-consuming scenario of cycle formation is an emission of *two* closed contours which are walked in the opposite directions (see Fig. 6(b) on p. 28: the there-and-back-again path visits its starting point thrice and is torn exactly in the middle, which creates two mutually inverse replicas; energy is spent on the disruption of the synonym of trivial word $\langle 1\rangle = \langle a\cdot a^{-1}\rangle$ in two nontrivial cyclic words $\langle a\rangle$ and $\langle a^{-1}\rangle$.

We postulate that *particles* are the meaning of cyclic words that encode contours along the graph; such words contain the mark-up of contracted edges (this has nothing to do with the contractions configuration for edges where the path does not run). We have noted in the previous section that the dynamics of contraction configurations determines the evolution of curvature and hence mass-energy. We postulate that this is the mass endowment mechanism for particles: if there is at least one contracted edge along the contour, the particle is *massive*; otherwise it is *massless* (see Fig. 5 on p. 26). Note that a given continuous contour with its contractions mark-up could have different masses with respect to the continuous limits of different tilings of space.

To formalise this approach in algebraic terms and make applicable the formalism of [7], we say that a *particle* is a functional

$$\mathcal{H} = \sum_{\langle \mathrm{words}_{\boldsymbol{x}}\rangle} \int \langle \mathrm{cycle}_{\boldsymbol{x}}\bigl(\mathbb{S}^{\pm 1}, \vec{\mathrm{x}}_i^{\pm}\bigr)\rangle \,\mathrm{d}\mu(\boldsymbol{x}),$$

where we use the following notation: The cycle is a closed contour that starts and ends at a point $\boldsymbol{x}$ of *continuous* space (a reference of the particle to a point $\boldsymbol{x}$ is equivalent to referring it to any other point $\boldsymbol{y}$ which lies on the contour passing through $\boldsymbol{x}$; still this amounts to a replacement of the countour's cyclic word by its equivalent, now starting the walk at $\boldsymbol{y}$, which leaves the contour intact); the *measure*, which refers to sets of points in space but does not exploit the notion of length, allows us to refer the particle to just one point – or create a cloud of matter by spreading the contour in space over a given set;[20] the units of measurement for the functional $\mathcal{H}$ are those of energy.

[20] A typical macroscopic diameter of such set of reference points would be the diameter of proton, which is $\approx 10^{-15}\,\mathsf{m}$, making $\sim 10^{18}$ Planck units.

Remark 3.1. In terms of the diffeo-class geometry of [7], the functionals $\mathcal{H} = \sum_i \int h(\boldsymbol{x}, t)$ $\langle \text{word}_i \rangle \, \mathrm{d}\boldsymbol{x}$ are *Hamiltonians* defined on the total space of the noncommutative tangent bundle over space, or on the total space of the infinite jet bundle over such bundle. The *values* of such functionals are (formal sums of) possibly massive contours; particles interact by using algorithms and structures which we discuss in sec. 4 (see [7, 6] and also [8] for a diffeo-class, commutative version of the setup). Interactions between particles form a chain of events which counts the time for a local observer.

If a particle is referred to only one point of space, the discrete measure realises the Dirac delta-distribution; note that it is the information about the contour and its properties which is ascribed to one point — still nothing is "compressed." In particular, it is impossible to split an electron in fragments and assemble it by trasporting such fragments from the spatial infinity to a given point, spending an infinite energy to overcome the repulsion potential of the would-be fractions of the charge $-e$; this approach also resolves the difficulty with an infinite density of electron's mass. Indeed, the particle is proclaimed existing at a certain point of space.

On the other hand, for dimensionful particles like proton or neutron the measure is concentrated on a larger set; typically, its support is a connected bounded set. We thus exclude from further consideration rapidly descreasing distributions with –juridically speaking, unbounded– supports.

Remark 3.2. The interactions operate with the *values* of the functionals that encode particles but not with the functionals themselves; in effect, particles interact as indivisible entities so that the processes refer to the particles' existence but not to their instant "shapes," for those are undefined due to the Heisenberg uncertainty principle. For example, in the course of decay $\mathsf{n}^0 \longmapsto \mathsf{p}^+ + \mathsf{e}^- + \overline{\nu}_{\mathsf{e}}$ the free neutron stops existing in space at all its points when proton and the other two particles are formed or start to form.

Remark 3.3. The edges which are contracted break the symmetry of any contour; they also produce the irregularity of the graph's structure near the merging vertices, not necessarily at the points of the contour. Such defects induce the particle to interact with other objects; conversely, massless point particles with simple contours without contracted edges or marked vertices (e.g., not carrying electric charge) demonstrate very low cross-sections for interaction with matter.

The scalar field of *zero-length* words is the density of vacuum energy. It does not show up in the form of energy communicated to any particles chiefly because of absence of those particles; it just is. However, we already know that the trivial word $\langle 1 \rangle$ is synonymic to paths $\langle a \cdot a^{-1} \rangle$ walked twice, there and back again; in particular, it is synonymic to a closed cycle a and then a^{-1} walked in the opposite directions. By spending some extra energy on disrupting the contour a from the anticontour a^{-1}, which creates two cyclic-invariant words of positive length, we convert a part of the spare vacuum energy into the *matter-antimatter* pair of particles.

3.2. **Matter versus antimatter.** The orientation of the CW-complex distinguishes between two directions to walk around a given contour (e.g., a face of a cell).[21] To pass a contour backwards, the relay is this:

- replace each letter $\mathbb{S}^{\pm 1}$ or $\vec{x}_i^{\pm 1}$ with its inverse, resp., $\mathbb{S}^{\mp 1}$ and $\vec{x}_i^{\mp 1}$;
- read the word backwards, i.e., in the right-to-left order.

This mechanism tells matter from antimatter; the same principle is applicable literally to formal sums of non-closed paths, see sec. 4.2. Thus, a distinction between matter and antimatter is conventional; yet the two anti-worlds can differ in their physical properties due to the CP-symmetry violation (see below).

Let us notice further that by pasting a word that means matter at the beginning or at the end of the respective word for antimatter, or *vice versa*, one obtains the trivial product $\langle 1\rangle$ of the two paths. This is why energy is released in the course of *annihilation*; such energy impulse can take the shape of a photon-antiphoton pair, etc. (see sec. 3.3.2); after its minor part is spent on the disruption of contours, the exact amount of released energy depends on the mass-energy of the two vanishing enantomorphs.

The fact that the CW-complex is oriented not only distinguishes between Left (L) and Right (R) but also motivates a possible violation of the CP-symmetry, which itself is the mirror-reflection Left $\rightleftarrows$ Right in the orientation of space *and* a substitution matter $\rightleftarrows$ antimatter (i.e., reading backwards all the words from the glossary).

Explanation. Let us take a –now-existing– tetrahedron in the spatial part of the graph and contract it along three bold edges as in Fig. 3, doing this in two mirror-symmetric ways (note that marking the vertices by digits is an act of will yet the two scenarios are true mirror copies of each other). Let us now recall that a catalogued particle itself and

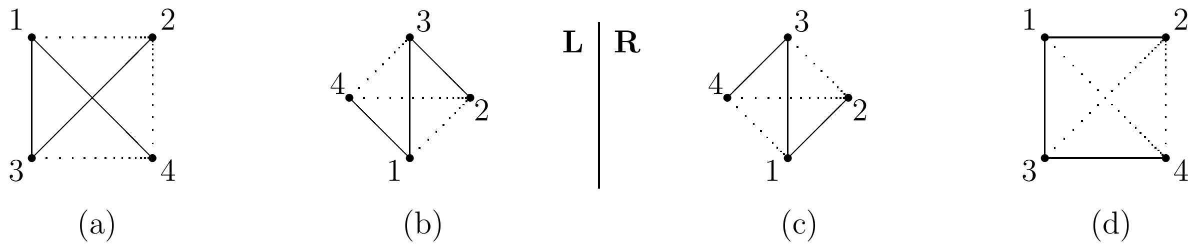

Figure 3. Mirror-symmetric contractions.

the processor which handles particles –e.g., by disrupting contours and reconfiguring the available edges– is an automaton: it reads the (cyclic) words from the glossary[22] and crawls along the contracted graph by running a program like this:

R: `move right, then right, then right again`

[21]We emphasize that the order in which one passes the edges when reading the contour's cyclic word is not the same as a choice of the contour's orientation (either matching or reverse with respect to the orientation induced from the CW-complex); we reserve that choice for the definition of spin.

[22]The catalogue of matter and antimatter is the glossary of equivalence classes of contour-determining cyclic words with a mark-up of the edges to-contract; the glossary is independent from the actual configuration of contractions in the graph — it is indeed a list of words.

on the plane containing Fig. 3(d). Under the instant CP-transformation, the automaton is interrupted and starts the relay program[23]

L: `move left, but before that move left, still before which move left`

on the plane in Fig. 3(a).

Suppose for definition that the tetrahedra in Fig. 3(b-c) are *the only* contracted edges in the Universe and it is these two objects which encode the choice of its orientation. The mechanism of CP-symmetry violation is that the mirror-symmetric contractions of the tetrahedron produce unequal configurations of the tadpoles, see Fig. 4. Thus,

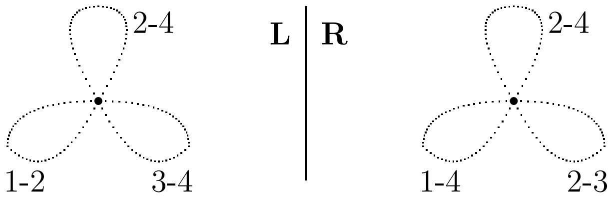

FIGURE 4. Formation of tadpoles in Fig. 3.

the R-automaton that runs the R-program and, for definition, reaches the vertex $\underline{1}$ after its first step terminates the program at the point two steps to the right from the vertex $\underline{2}$. Now reshape the tetrahedron's contraction and switch to the antiparticles and L-program; the L-automaton starts at the R-automaton's endpoint, runs the L-program, and terminates at the point $\underline{1}$ (but *not* at the starting point of the R-automaton) because the edge $\underline{2} \to \underline{1}$ becomes a tadpole in the antiworld instead of being contracted in the right-oriented world (the orientation is determined by the order $1 \prec 2 \prec 3 \prec 4$ in Fig. 3(c)).

In particular, it may happen that by running its R-program the R-automaton crawled along the *closed* contour while reading the right words from the glossary but the path of the L-automaton appears to be *not closed*, after the orientation was reversed by switching from Fig. 3(c) to Fig. 3(b). Recall that a reaction is information-processing the output of which is a word, and the Left and Right automata attempt to create the particles' countours by reading this word in one of the two possible directions. We conclude that, specifically to the scenario which we have discussed, the L-channel is either altogether suppressed (by an earlier convention that spatial edges are fermionic) or is possible at an expense of energy needed to contract extra edge(s) in order to close the L-path and thus bring the L-automaton to its starting point.

This produces the CP-asymmetry if the preference of Right over Left is implanted in the Universe since the moment of decontraction of the first tetrahedron in its history. □

Corollary 5. The *masses* of particles and respective antiparticles can be (slightly) unequal.

Corollary 6. If Right has been prevailing over Left since the moment when creation of particles became possible, there is an imbalance between matter and antimatter nowadays.

[23]Notice that the order of reading letters in catalogued words and the arising precedence "before" and antecedence "after" have nothing to do with the time as physical process; it is not appropriate to postulate that antimatter flies backwards in time.

Remark 3.4. The risk of CP-symmetry violation is built into the automaton which (1) reads from the glossary dogmatically and (2) attempts, by disrupting a word in between two letters and by having disrupted another given cyclic word, to paste the *open* word from the glossary in between, or immediately before or after the letters of the given word, and then (3) attempts to realise the output as a route along the graph — not taking into account the actual configuration of contractions.

On the contrary, the functionality of a different-type automaton that handles already-existing closed contours in space is stable. Indeed, the already paved routes remain closed if they had this property before the CP-transformation (note that the decontracting edges are pasted into the contours), see sec. 4.2 for details.

3.3. **Properties and examples of elementary particles.** The definition of quantum numbers and each act of their measurement channels information between tiling(s) of the homeo-class topological manifold by a CW-complex and the diffeo-class, smooth and commutative visible space. There is no surprise that the available data which can be transmitted in this way must be very rough and appeal to topological invariants only.

3.3.1. *The spin.* From now on let us suppose that the contours which determining particles are *oriented*; note that a choice of orientation for the contour's edges within each of its loops (e.g., consider a bouquet of circles) is in general not correlated with the order in which these edges are passed when one reads the particle's cyclic word.[24] We notice also that a contour's orientation is thus not necessarily inherited from the orientation of the CW-complex.

Definition 5. The *spin* of a given particle is a quantum number which is equal to the sum

$$s = \sum_{\text{loops}} \pm\tfrac{1}{2}\hbar$$

over all the loops in the spatial component of the closed contour. (The tadpoles $\mathbb{S}^{\pm 1}$ at each vertex are dealt with separately in the next section). The contribution of a loop is $+\frac{1}{2}\hbar$ if the choice of its orientation coincides with the order in which these edges are written in the cyclic word, and equals $-\frac{1}{2}\hbar$ otherwise.

Almost all known particles do have spin and most of the stable particles (either massive or without mass) are coded by simple contours –the unknots– so that their spins are $\pm\frac{1}{2}\hbar$.

Remark 3.5. The orientation of a loop in a contour can instantly switch from $+\frac{1}{2}\hbar$ to $-\frac{1}{2}\hbar$, making no effect on the orientation of other loops in the contour, so that the overall value of the spin changes by unit steps $\pm\hbar$ between its minimal nonpositive and maximal nonnegative values.

Remark 3.6. In view of a possibility for an sudden swap of orientation of a given loop, it is still meaningful to say that products of a weak reaction –in the course of which the contours are instantly disrupted and recombined– can in that moment retain the

[24] Experiments report that the electron neutrinos ν_{e} usually have *left helicity*, that is, the orientation of their contours makes left-handed helix propagating in the direction of their macroscopic instant velocity; likewise, the electron *anti*neutrinos $\overline{\nu}_{\mathsf{e}}$ tend to have right helicity, see Fig. 5 on p. 26.

orientation of edges from loops in the input particles. We therefore expect that the spins of free neutrons and protons which are emitted in the β-decay $\mathsf{n}^0 \longmapsto \mathsf{p}^+ + \mathsf{e}^- + \overline{\nu}_\mathsf{e}$ *are* correlated (though for none of them we can measure for certain the projection of spin to a given direction in the macroscopic space at the moment of decay, and though neither before nor after the reaction two measurements of the same particle's spin would necesserily coincide).

3.3.2. *The electric charge.* Let us recall from sec. 2 that each vertex of the CW-complex (and every point in the continuous limit of a tiling) carries the tadpole(s) $\mathbb{S}^1$ starting and ending at that point.

Definition 6. The *electric charge* of a given particle is a quantum number which is equal to the difference

$$c = \sum_{\langle\text{words}\rangle} \bigl(\sharp\,\mathbb{S}^1 - \sharp\,\mathbb{S}^{-1}\bigr) \cdot e$$

of the numbers of time the particle's contour passes the tadpoles –as they are written in the particle's word(s)– in positive and negative directions, each loop in the compactified dimension thus contributing with the respective elementary charge $\pm e$.

Corollary 7. The electric charges of particles and their antiparticles are equal by absolute value and have opposite signs.

Remark 3.7. In the beginning, the initial electric charge of the Universe was equal to zero because the contours did not exist yet (for all the tadpoles were contracted).

Corollary 8. Nowadays, the Universe is overall electrically neutral because all the separately existing electric charges in it were obtained by disruption of neutral contours (indeed, all the contours were obtained from copies of the trivial word $\langle 1\rangle$ by decontraction of edges, see Fig. 2 on p. 20, or by disruptions, see Fig. 6 on p. 28).

Remark 3.8. There is a temporary shortage of the *magnetic* charges in this Universe.

3.3.3. *The photon.* By introducing the following definition we resolve a delicate issue that the photon is a bosonic particle that can exist in two possible states (or *polarisations*), which usually requires that one manually removes the spin-zero state from the triplet $s \in \{-\hbar, 0, \hbar\}$.

Definition 7. The *polarised photons* γ are massless point particles whose cyclic words are

$$\begin{aligned} \gamma_{\circlearrowleft} &= \langle \mathbb{S}^1_m \mathbb{S}^{-1}_n \rangle, \\ \gamma_{\circlearrowright} &= \langle \mathbb{S}^{-1}_m \mathbb{S}^1_n \rangle, \qquad m < n, \end{aligned}$$

where $m, n \in \mathcal{I}$ belong to the ordered indexing set at a point of quantum space; photons carry no electric charge ($\pm e \mp e \equiv 0$).

Corollary 9. The polarised *antiphotons* $\overline{\gamma}$,

$$\begin{aligned} \overline{\gamma}_{\circlearrowleft} &= \underrightarrow{\langle \mathbb{S}^1_m \mathbb{S}^{-1}_n \rangle}^{-1} = \underrightarrow{\langle \mathbb{S}^1_n \mathbb{S}^{-1}_m \rangle} = \langle \mathbb{S}^{-1}_m \mathbb{S}^1_n \rangle = \gamma_{\circlearrowright}, \\ \overline{\gamma}_{\circlearrowright} &= \underrightarrow{\langle \mathbb{S}^{-1}_m \mathbb{S}^1_n \rangle}^{-1} = \underrightarrow{\langle \mathbb{S}^{-1}_n \mathbb{S}^1_m \rangle} = \langle \mathbb{S}^1_m \mathbb{S}^{-1}_n \rangle = \gamma_{\circlearrowleft}, \qquad m < n, \end{aligned}$$

are identical to the photons with opposite polarisations.

Remark 3.9. Photons travel[25] in space with invariant light speed c. However, when a photon is stopped by a material object in *continuous* space at a given point, the indexed set of fermionic tadpoles at that point reduces to a unique circle $\mathbb{S}^{\pm 1}$ so that the photon $\langle \mathbb{S}^{\pm 1}\mathbb{S}^{\mp 1}\rangle$ immediately becomes synonymic to the zero-length word $\langle 1\rangle$. Thus, photons γ play the rôle of energy carriers.

3.3.4. *Lepton–neutrino matchings.* Let us use the tiling associated with the root system A_3; this prescribes the configuration of local information channels for the rules which process information about particles in the course of reactions.

Definition 8. The *electron* e^-, antielectron (or *positron*) e^+, *electron neutrino* ν_{e}, and *electron antineutrino* $\overline{\nu}_{\mathsf{e}}$ are point particles whose contours are drawn –schematically and up to homeomorphisms– in Fig. 5; the contracted edges are marked by dotted lines. Each of the four particles has spin $\pm\frac{1}{2}\hbar$ due to the two possibilities to choose an

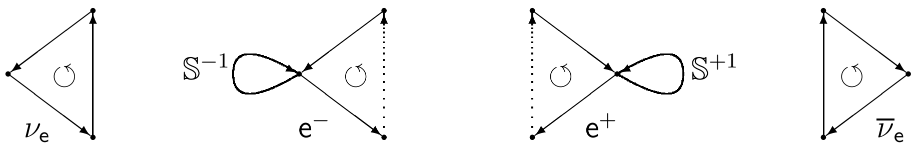

FIGURE 5. (Anti)electron and electron (anti)neutrino.

orientation of these unknots in $\mathbb{E}^3$. The electron e^- has negative electric charge $-e$ and that of positron e^+ equals $+e$.

Corollary 10. Apart from carrying no electric charge and probably having no mass, the electron (anti)neutrino is in all other aspects “indistinguishable” from the (anti)electron.

The mass of (anti)electron is approximately 0.511 MeV/c^2; by using Fig. 2 on p. 20 we argued that there *are* chargeless massless spin-$\frac{1}{2}\hbar$ particles and we now identify them with electron neutrinos; however, our reasoning does not forbid the existence of particles with very similar properties yet with a tiny mass (hence travelling slower than light). If being massless and thus having no naturally marked vertex or edge –unlike the (anti)electron– neutrinos symptomise a general disinclination to interaction of any kind with any type of matter.

However, we owe almost everything to these simplest neutrinos because it was the contour ν_e which was first created in the spatial component of the CW-complex in the course of decontraction of the first oriented face, see Fig. 2; the first photon γ appeared at about the same time, after the decontraction of the first tadpole; we see no reason to debate which event happened earlier.

The two heavier point leptons are the (anti)*muon* $\mu^\pm$ ($\mathsf{m}_\mu \approx 105.7$ MeV/c^2) and (anti)*tau* $\tau^\pm$ ($\mathsf{m}_\tau \approx 1776.8$ MeV/c^2); these four particles are unstable (their proper life-times are approximately $2\,\mu\mathsf{s}$ and $3\cdot 10^{-13}\,\mathsf{s}$, respectively); in the course of decay they are reported [12] to produce the respective (anti)neutrinos ν_μ, $\overline{\nu}_\mu$, ν_τ, and $\overline{\nu}_\tau$, which is quite

[25]In sec. 2.2 we understood the propagation of a photon as the automaton that creates unit events by destroying the photon at one end of an edge in the graph and creating the same photon at the other end. The count of such events is the pace of time.

logical: after the decontraction of heavy leptons' contours, their electric charges are decoupled –by spending a minor part of the released energy on detachment of the charge tadpoles– and re-attached to the newly-generated and separated mutually inverse pairs of contours for lighter leptons and their antineutrinos.

Example 3.1. Consider the Michel decay $\mu^- \longmapsto \nu_\mu + \mathsf{e}^- + \overline{\nu}_\mathsf{e}$. The decontraction of edges which endow the muon μ^- with mass and deprivation of its contour from its negative electric charge, which is encoded by the tadpole $\mathbb{S}^{-1}$, produces the neutrino ν_μ and a store of dispensable energy, which has been partly used to detach the charge and which is used then to disrupt the trivial word issued from the point where the charge is located,

$$\langle 1\rangle = \langle \nu_\mathsf{e} \cdot \overline{\nu}_\mathsf{e}^{-1}\rangle \longmapsto \langle \nu_\mathsf{e} \cdot \mathbb{S}^{-1}\rangle + \langle \nu_\mathsf{e}\rangle.$$

A contraction of edge(s) in the charged first term of the right-hand side yields the second and third particles in the reaction's output. The remainder of energy (left from the excess of muon's mass) is communicated to the three particles ν_μ, e^-, and $\overline{\nu}_\mathsf{e}$ as their kinetic energy. (Note the overall conservation of the electric charge and a likely preservation of the muon's spin by the muon neutrino ν_μ.)

However, we see that the discarded contour for ν_μ in Michel's reaction and the mutually reverse prototypes for e^- and $\overline{\nu}_\mathsf{e}$ could co-exist on *different* CW-complexes which fill in a unique continuous homeo-class Universe; we notice that the trivial word $\langle 1\rangle$ to-expand at the location of the charge does not refer to a choice of the alphabet, whereas the store of energy and the loop along $\mathbb{S}^{-1}$, not leaving the point of physical space, are logically transportable between the schemes of information processing.

If so, the decays of the heaviest $\tau^\pm$ and less heavy $\mu^\pm$ could be second-order phase transitions, in the course of which the information channels are locally reconfigured (i.e., one crystal structure transforms into another –no less well ordered– so that the order parameter is constant) and the energy and electric charge are pumped into the new logical processor of information; the old processor calculates the receding of muon's neutrino.

Conjecture 11. There *are*, and there are exactly *three* types of leptons –e, μ, and τ– because there are exactly three types of irreducible lattices –A_3, B_3, and C_3– in Euclidean space $\mathbb{E}^3$, open domains in which are homeomorphic to the spatial components of domains in homeo-class realisation of the Universe.

4. Geometry of fundamental interactions

Let us finally focus on the algebraic structures of the four fundamental interactions; we assume for definition that the processes of reconfiguration and interaction occur on the same lattice so that the alphabet $\vec{\mathrm{x}}_i$, $\mathbb{S}^1 = \overrightarrow{\mathrm{tt}}$ is common for all particles. We first consider the weak and strong forces and then we discuss the long-range electromagnetism and gravity.

We recall that the algebraic operations which we usually do with words are

- writing them consecutively,
- disrupting a word in between two letters in accordance with the hyphenation rules.

Notice that the first option prevails in frequency over the second whenever a sufficiently long fragment of text is already written; however, Nature first constructed the hyphenation table for its glossary.

4.1. **The weak force.** The defining property of a weak process by which it is recognised at once is an arbitrary combination of the following very unlikely events:

- a zero-length word, which means just one vertex in terms of paths, expands to a synonymic *cycle* which is walked twice in opposite directions;
- a path, being either a previously existing contour from the reaction's input or a newly-produced synonym of $\langle 1\rangle$, is torn.

Then the available collection of paths' fragments, which themselves may be not cyclic words but their separate letters or syllables, recombine and join the loose ends, forming new countours and thus creating new particles. Note that the output of a weak reaction is an anagram of the letters belonging to the original text — which was extended with extra letters and their negations by using the add-subtract arithmetic trick.

Example 4.1. Let us consider the weak decay of a free neutron n^0; having spin $\pm\hbar/2$ and parity $+1$ (i.e., the orientation matching of the unknot's word and the orientation of the lattice is "yes"), the neutron's contour resembles the one drawn in Fig. 6(a). The

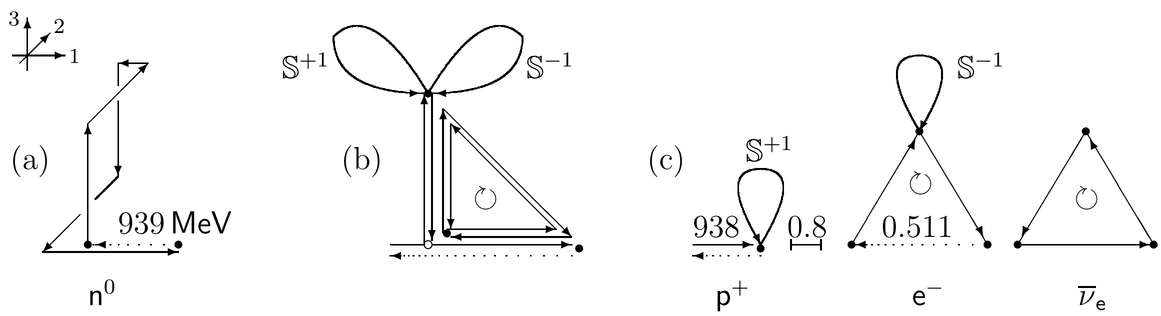

FIGURE 6. Decay of free neutron.

primary channel ($\approx 99.9\%$) of the β-decay is

$$\mathsf{n}^0 \longmapsto \mathsf{p}^+ + \mathsf{e}^- + \overline{\nu}_{\mathsf{e}} + \langle 0.8\,\mathsf{MeV}\rangle. \tag{1}$$

The reaction is energy-positive: the difference in mass between n^0 and the proton p^+ exceeds the mass of electron; our previous argument in sec. 3 (see Fig. 2) shows that the electron (anti)neutrinos ν_{e} and $\overline{\nu}_{\mathsf{e}}$ are massless. The energy which is stored in the neutron's contracted edge(s) has the traditional choice of three fates: to remain stored in the proton's mass, to endow the electron with mass, or to be spent on the disruption of contours and exchange of edges; the rest is communicated to the three emitted particles as their kinetic energy.

The geometry of cross reactions refers to the same Fig. 6(b-c):

$$\mathsf{p}^+ \longmapsto \mathsf{n}^0 + \mathsf{e}^+ + \nu_{\mathsf{e}} \qquad \text{(inverse } \beta\text{-decay)},$$

$$\mathsf{p}^+ + \mathsf{e}^+ \longmapsto \mathsf{n}^0 + \nu_{\mathsf{e}} \qquad \text{(electron capture)}.$$

Another channel ($\approx 0.1\%$) of a free neutron's decay is

$$\mathsf{n}^0 \longmapsto \mathsf{p}^+ + \mathsf{e}^- + \overline{\nu}_{\mathsf{e}} + \gamma.$$

This reaction inserts the null path $\langle 1\rangle = \langle \mathbb{S}^1\mathbb{S}^{-1}\rangle$ in between the factors which assign the opposite charges to p^+ and e^-:

$$\langle 1\rangle = \langle \underbrace{\mathbb{S}^1}_{+e} \cdot \underbrace{\mathbb{S}^{-1}}_{-e}\rangle = \langle \underbrace{\mathbb{S}^1}_{\mathsf{p}^+} \cdot \underbrace{\mathbb{S}^1\mathbb{S}^{-1}}_{\gamma} \cdot \underbrace{\mathbb{S}^{-1}}_{\mathsf{e}^-}\rangle.$$

The proportion 0.1% demonstrates how unwilling SATOR AREPO is to rotate twice not once around the circle in the compactified dimension (see Fig. 6(b)).

We conclude that the smooth complex $SU(2)$-gauge theory over the space-time viewed as a smooth manifold could have no direct relation to the Nature's prosaic effort in describing the weak interaction.

4.2. **The strong force.** The strong interaction does not beg, borrow, or steal the contours which did not belong to the reaction's input but which could be created at the expense of the zero-length words of energy; the output of a strong reaction looks much the same as the original text, only the order of sentences in it is mixed. For example, several protons and neutrons link to a bouquet or form a chain mail, that is, a nucleous. We emphasize that the strong interaction handles the already-existing contours (i.e., not just the words encoded in the glossary via route instructions and edge contraction configurations).

The binary algebraic operation that creates the strong force is the multiplication $\times$ for cyclic words; we described it in detail in [5, 7] (see also [9]). Essentially, this is the standard unlock-and-join technique of the topological pair of pants $S^1 \times S^1 \to S^1$. By definition, the value of operation $\times$ at two paths is calculated in three steps:

(1) both contours are unlocked at one vertex each;
(2) the unlocked paths are transported along the lattice such that the two (un)locks coincide;
(3) the loose ends of the disrupted contours are recombined in such a way that the left-to-right order of reading the words is preserved.

The structure $\times$ takes the sum over all possible (or preferred) locations of the locks on each of the countours; if a certain summand is forbidden (e.g., if oriented spatial edges of the graph are viewed as fermions), then it is omitted; the result is normalised by the actual number of contributing terms.

Notice that the multiplication of the particles' contours is commutative but not associative; indeed, the contours $|1\rangle$ and $|2\rangle$ are always adjacent in the product $\bigl(|1\rangle \times |2\rangle\bigr) \times |3\rangle$ but they can be separated by edges of $|3\rangle$ in some of the terms in $|1\rangle \times \bigl(|2\rangle \times |3\rangle\bigr)$. Yet we recall that the strong interaction is not associative at the level of nuclear fusion and fission: e.g., the channel $\bigl(\mathsf{p}^+ \times \mathsf{n}^0\bigr) \times \mathsf{p}^+ \longmapsto {}^3_2\mathsf{He}$ is not realised via $\bigl(\mathsf{p}^+ \times \mathsf{p}^+\bigr) \times \mathsf{n}^0$ by a would-be intermediate two-proton ${}^2_2\mathsf{He}$; likewise, the equally possible (50%-50%) processes ${}^2_1D + {}^2_1\mathsf{D} \longmapsto {}^3_1\mathsf{T} + \mathsf{p}^+$ or $\longmapsto {}^3_2\mathsf{He} + \mathsf{n}^0$ do not amount to a bare assembly of two protons and two neutrons.

Nevertheless, to grasp the associativity by the triangle equation,

$$\bigl(|1\rangle \times |2\rangle\bigr) \times |3\rangle = |1\rangle \times \bigl(|2\rangle \times |3\rangle\bigr),$$

one has to deform the commutative non-associative binary operation $\times$ to the associative but not commutative star-product $\star = \times + \text{const} \cdot \hbar \cdot \{\,,\,\}_{\text{Poisson}} + \overline{o}(\hbar)$, converting all the structures and operations at hand into power series in the Planck constant $\hbar$. This could be done by a proper upgrade of the deformation quantisation technique [8]; the Poisson bracket $\{\,,\,\}_{\text{Poisson}}$ of particles appears in the $\hbar^1$-slice of the full quantum picture.

Note also that the expected property of the structure $\star$ to be associative but not commutative implies the violation of the P-symmetry, which is (Left $\rightleftarrows$ Right) $\Longleftrightarrow$ $(|1\rangle \rightleftarrows |2\rangle)$, by the strong force.

Finally, we recall that the full power of noncommutative calculus is revealed by the introduction of noncommutative bundles π^{nC} whose base is the noncommutative tangent bundle over space, i.e., the homeo-class space itself (see [6, 7]). Namely, let us consider *open*, positive-length words (containing the mark-up of contracted edges) which we view as noncommutative fields over space; the new structures are auxiliary in the description of the full quantum geometry of the strong force and may determine no particles existing as independent objects, yet they could be helpful. These fields are sections of the noncommutative bundle π^{nC} over the homeo-class space. At each point of the base, a local basis of such fields extends the alphabet $\vec{\mathrm{t}}^{\pm 1}$, $\vec{\mathrm{x}}_i^{\pm 1}$ of generators of the lattice; not without insight we denote by $(\mathsf{u}, \mathsf{d}, \mathsf{s}; \mathsf{c}, \mathsf{t}, \mathsf{b})$ the elements of such bases introduced pointwise.[26] We foresee that the supports of those (anti)sections are finite in space in all inertial reference frames yet such supports do not amount to single points but contribute to the construction of clouds of matter, that is, dimensionful particles.

Corollary 12. Provided that the information encoding leptons is referred to one point in space at each instant of time, and under the hypothesis that the sections of π^{nC} are piecewise-continuous, the leptons may not consist of such auxiliary building blocks.

Remark 4.1. Because the auxiliary blocks $(\mathsf{u}, \mathsf{d}, \mathsf{s}; \mathsf{c}, \mathsf{t}, \mathsf{b})$ are introduced to encode formal sums of *open* paths, they in practice can not be isolated physically and registered as objectively existing particles. (The same argument applies now, long after the Big Bang, to each generator $\vec{\mathrm{x}}_i$ of an aperiodic affine lattice in $\mathbb{E}^3$.)

4.3. **Electromagnetism.** Now we address the effective long-range interactions: electromagnetism first and then gravity. It must be noted that both concepts involve the same idea of the edge contraction (in disguise, formation of tadpoles), that is, a prescription that the endpoints of an edge become one vertex. (The difference between the two concepts is that a spatial edge vanishes altogether, not forming a loop, whereas the generators $\mathbb{S}^{\pm 1}$ or $\vec{\mathrm{t}}^{\pm 1}$ are tadpoles for granted.) Therefore, it is logical to expect that the macroscopic properties of the two forces are much alike in their classical description: a charged massive point locally produces the Newtonian electric and gravitational potentials inverse-proportional to distance in space.

Remark 4.2. The count of electric charge, which is by definition the difference $\sharp\mathbb{S}^{+1} - \sharp\mathbb{S}^{-1}$ of the numbers of loops that a path winds in the fourth, tadpole dimension, does not interfere with the disruptions or rearrangements of the path's spatial edges.

[26] As usual, the inverse elements $(\overline{\mathsf{u}}, \overline{\mathsf{d}}, \overline{\mathsf{s}}; \overline{\mathsf{c}}, \overline{\mathsf{t}}, \overline{\mathsf{b}})$ mean the inversions $\vec{\mathrm{t}}^{\pm 1} \mapsto \vec{\mathrm{t}}^{\mp 1}$, $\vec{\mathrm{x}}_i^{\pm 1} \mapsto \vec{\mathrm{x}}_i^{\mp 1}$ and reading the summands backwards in each component of a section for π^{nC}.

Consequently, the electric charge is conserved but in other aspects it does not influence the weak or strong processes.

The large-scale alikeness of the long-range forces is readily seen from the coding of electron on the equilateral triangular lattice, see Fig. 7: the particle consists of

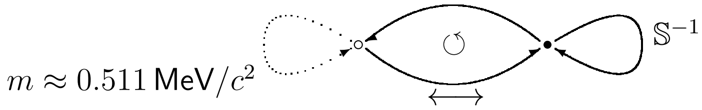

FIGURE 7. The electron e^-.

indivisible mass and indivisible charge which are held close to each other but always stay separated by emptiness; the information about the particle's closed contour does not take shape of a faintly shimmering rope or cord. Note that the contraction of the edge which endows electron with mass creates the asymmetry of space surrounding the point where the mass is located; on the other hand, the formation of the negative charge $-e$ by the tadpole $\mathbb{S}^{-1}$ is fully symmetric with respect to space.

Remark 4.3. It is also logical that, whenever accelerating in electromagnetic field, the electron *radiates*. Indeed, Lorentz' force acting on its charge, the electron becomes an oscillator in which space itself plays the rôle of elastic spring between the point charge and the point mass; the oscillations then amount to periodic elastic deformations of the space-time structure, pulling or slowing the mass as it retards or overtakes the charge.

In conclusion, electromagnetism is a very famous example of a cyclic-invariant theory; outside point particles, its gauge description could be *exact*.

4.4. **Gravity: the Big Bang logistics.** [27] Let us attempts to track logically the scenario of events in the early Universe, taking into account the assertions which we have made so far.

1. Initially, the Universe consisted (according to its topology) of only one point, which is encoded by the statement that the CW-complex was fully contracted, thus formed by only one vertex and no edges. It is possible that the initial point was also assigned an extra number that indicated the energy surplus over its store in the contracted edges.

2. The fully contracted lattice was released from hold and its edges began to decontract; each event of edge decontraction released energy which took shape of the zero-length word $\langle 1\rangle$. The time started; it first amounted to the count of decontraction events and derivative events of reconfigurations in the lattice defects' portrait; still no particles were formed yet and there was no light.

A decontraction of the first spatial edge with tadpoles at each of its ends created the possibility of existence of light viewed as the automaton that propagates the null photon's path $\langle\mathbb{S}_m^{\pm 1}\cdot\mathbb{S}_n^{\mp 1}\rangle$ from a vertex to its neighbour; this does not imply that the ready-to-work automaton did actually start to work at once. The decontraction of the first triangle did create the first electron neutrino (see Fig. 2); we think that it is

[27] We owe this term to Yu. I. Manin; let us also recall that the ordering of the Universe in small boxes and events which led to the Big Bang are depicted in the Russian edition of [11] on p. 268.

scholastic to debate whether the first photon preceded the first neutrino or *vice versa*. For the first tetrahedron to decontract, a coin was thrown and this determined whether that was a Left or Right tetrahedron (see Fig. 3); space was thus oriented and Nature chose Right.

The dimension of the CW-complex became positive, and the initial point split to a set of points (depending on the convention about homeo-class continuous space or quantum space as a lattice, to continua or to finite sets through a finite number of decontraction events, respectively); space began to expand.

3. While the edges kept on decontracting in the replicas of adjacency table for each newly-produced copy of the original vertex, such events became independent and uncorrelated. Because of this, the values of noncommutative Ricci and scalar curvature were (almost) random at points of the early Universe.[28] However, let us recall from [10] that in this case the Jacobi field connecting two infinitesimally close geodesics issued from a point grew exponentially — yet, paradoxally, for almost certain there appeared an arbitrarily large number of those geodesics' focus points, which resulted in clashes and information exchange. (Here we use the assumption of space's continuity and use light to introduce the smooth structure of space at the expense of infinite energy.) The Universe experienced the *inflation*.

4. It took certain time for space to expand and reach a configuration with a relatively small fraction of edges which remained contracted in large finite neighbourhoods of many points. (Because the Universe continues expanding *now*, we may not refer to all of its points but operate with sufficiently spacious regions.) Simultaneously, those lattice defects could assemble due to the reasons of entropy. However, the preceding decontraction of a major part of edges in those domains released a colossal amount of vacuum energy; it shaped into a scalar field over space.

5. Although the initial point of the Universe split (its descendants still continue splitting at the outer periphery of the Universe, sending us photons and neutrino flows; throughout Cosmos, the edges between its descendants continue splitting and so contribute to the validity of Hubble's law), there remained or there re-appeared sets of "conservative-minded" vertices in the lattice which proclaimed themselves one-point-forever and merged the adjacency tables — but their neighbours refused to join the coalition. This created the singularities of the first generation of black holes. (Those were the times when a lack of curvature necessary to form the event horizon was out of dispute.) The eldest black holes produced considerable irregularities of the space(-time) geometry, which triggered the formation of matter from an otherwise still meta-stable state of the Universe already filled with neutrinos and photons.

6. The excess of vacuum energy, the decontractions which created massless neutrino cycles as in Fig. 2, photons, and possible entropy-based gradient flows of the dark matter caused the formation of elementary particles via weak processes of contour disruption and reconfiguration (in particular, relatively near –in cosmic sense– to black holes). The Left-Right asymmetry, which had been built into the Universe since the decontraction of the first tetrahedron, implied the domination of matter over antimatter in the course of particle creation. The add-subtract mechanism for creating, distributing, and counting the electric charge kept the initially pathless and hence chargeless Universe neutral *en masse*.

[28]The worst idea here would be that of *averaging*, see [10].

7. The world became what we know it now; only a small part of its mass and energy shaped into particles. Nevertheless, those were enough to form galaxies around the eldest black holes which called that matter from unbeing. The start of chain reactions of nuclear fusion and ignition of the first star heralded the end of the beginning in the history of the Universe.

Conclusion

We have outlined a possible axiomatic quantum picture of the fundamental interaction; we hope that it will help us to resolve a part of known difficulties or at least offer us a good reformulation of paradoxes in the existing paradigm.

We have shown that the chosen set of postulates implies the following statements:
1. At (sub-)Planck scale, gauge theory is insufficient for a description of the interactions; one should use a theory that does not appeal to the locally-linear, diffeo-structure of the space-time but operate with geometry of the Universe at the topological, homeo-class level.
2. Vacuum, i.e., a domain in space which is known to contain no particles of any kind, can have mass-energy and, via its curvature, nonetheless produce gravity force; moreover, such vacuum does contain a store of energy which can be released and transmute into (anti)matter.
3. The (anti)matter is the meaning of information which is stored in space (specifically, within its topology and the loops paved through its homeo-realisation).
4. Having begun to expand from its initial state with the topology $\mathcal{T} = \{\varnothing, \text{Universe}\}$, the early Universe experienced an inflation phase of exponential growth.
5. The Universe is electrically neutral. In the quantum world, electromagnetism does contribute to the processing of information with the quantum number *charge* but plays no dominant rôle in the interactions and decays (e.g., protons and neutrons form the nuclei of atoms). Outside the particles and under the *ad hoc* assumption of differentiability for the gauge transformations, the $U(1)$-gauge model is *exact*.

We also logically argued in favour of the following possibilities:
6. The CP-symmetry violation in weak processes is a consequence of the Left $\neq$ Right asymmetry which has been being built into the Universe since it became oriented; matter now prevails over antimatter. The masses (whenever both are nonzero) of the respective (anti)particles can be slightly unequal.
7. The cosmic microwave background radiation is an immanent property of space itself, so it can not be shielded altogether by using any macroscopic medium.
8. The polarised antiphotons are cross-identical to the respective photons.
9. The weak processes are much less likely to occur than the strong ones.
10. A decay of the unstable leptons $\mu^{\pm}$ and $\tau^{\pm}$ and the (anti)neutrino oscillations $\nu_{\tau} \leftrightarrow \nu_{\mu} \leftrightarrow \nu_{\mathsf{e}}$ are second order phase transitions.

In conclusion, it is quite remarkable that matter *is*, even if it constitutes only about 16% of the should-be mass (and possibly 4% of the mass-energy) of the visible Universe. The main store of mass is contained in the dark matter, i.e., in the graphs' contracted edges where no nontrivial contours run, and there is a tremendous store of the vacuum energy in zero-length words and their synonyms.

We conjecture that the organisation of (anti)matter in galaxies around very massive objects, as we observe it now, summarises the history of the Universe itself. Namely, we view the singularities of black holes in the centres of galaxies as the eldest remnants of the Universe's One Point that did not expand to continua but served, and do so presently, as those inhomogeneities for the near-by regions of space which catalysed the creation of matter, i.e., release of contracted edges and formation of cycles (the principle being similar to boiling or condensation on admixtures). Thus, the presence of a black hole in the centre of each galaxy is not their fatality but the primordial blessing for their existence that called from unbeing the matter in them and then shaped it. For the same reason, it is unlikely that there are many lone stars outside galaxies.

Likewise, we expect that the giant voids in the large-scale structure of the Universe, the voids bounded by the walls formed by galaxies, are relatively poor in black holes and therefore are presently filled with the still dark matter and vacuum energy (however, in meta-stable state). This is why the voids look so empty even if they are so massive. Nevertheless, the superclusters, filaments, and walls in the large-scale organisation of the Cosmos stem from a topological configuration of vertices, edges, and cells in Planck-scale neighbourhoods of all its points.

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References

[1] Ambjorn J., Jurkiewicz J. and Loll R., Causal dynamical triangulations and the quest for quantum gravity, *Foundations of space and time* (G. Ellis, J. Murugan and A. Weltman eds., Cambridge Univ. Press., to appear), 19 p. arXiv:1004.0352 [hep-th]

[2] Cattaneo A.S. and Felder G., A path integral approach to Kontsevich quantization formula, *Commun. Math. Phys.* **212**:3 (2000), 591–611.

[3] Humphreys J.E., *Introduction to Lie algebras and representation theory*, Grad. Texts in Math. **9**, Springer-Verlag, NY, 1978.

[4] Kac V.G., *Infinite-dimensional Lie algebras*, Cambridge Univ. Press, Cambridge, 1990.

[5] Kiselev A.V., On the variational noncommutative Poisson geometry, *Physics of Particles and Nuclei* **43**:5 (2012), 663–665, arXiv:1112.5784 [math-ph]

[6] Kiselev A.V., *The twelve lectures in the (non)commutative geometry of differential equations*, 2012, 140p; preprint IHÉS M-12-13.

[7] Kiselev A.V., The calculus of multivectors on noncommutative jet spaces, Proc. int. workshop 'Noncommutative algebraic geometry and its application to Physics' (March 19–23, 2012; Lorentz Centre, The Netherlands) (2012), 16 p. arXiv:1210.0726 [math.DG]

[8] Kontsevich M., Deformation quantization of Poisson manifolds. I. *Lett. Math. Phys.* **66**:3, (2003), 157–216, arXiv:q-alg/9709040

[9] Kontsevich M., Formal (non)commutative symplectic geometry, 1993, *The Gel'fand Mathematical Seminars, 1990-1992*, Birkhäuser Boston, 173–187.

[10] Lamburt V.G., Sokolov D.D. and Tutubalin V.N., Jacobi fields along a geodesic with random curvature, *Math. Notes* **74**:3-4 (2003), 393–400.
[11] Manin Yu.I., *Mathematics as metaphor*, AMS, Providence, RI, 2007.
Manin Yu.I., *Matematika kak metafora*, 2nd ed. ext., Moscow, MCCME, 2010 (in Russian).
[12] Okun L.B., *Quarks and leptons*, North-Holland Publ., Amsterdam, 1980.
[13] Olver P. J., *Applications of Lie groups to differential equations*, Springer-Verlag, New York, 2000.
[14] Olver P.J. and Sokolov V.V., Integrable evolution equations on associative algebras, *Commun. Math. Phys.* **193**:2 (1998), 245–268.